Ask, don't just take: Property rules are more efficient than liability rules under asymmetric information

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Abstract: Liability rules allow unilateral takings against monetary compensation. This gives them an efficiency advantage over property rules when transaction costs impede a consensual transfer of entitlements. But property rules could still be superior: We show that transaction costs themselves depend on the mode of entitlement protection. In our model with private information about the owner's valuation and costly enforcement, only a property rule achieves the first best when the owner of the entitlement has all the bargaining power; in the opposite case with a take-it-or-leave-it offer from the potential taker, a property rule is more efficient than a liability rule for most parameter values. Welfare losses result from both misallocation of the entitlement and conflict costs. While liability rules can overcome bargaining impasse, they hamper the exchange of information and raise the cost of voluntary transactions.

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I. Introduction

Liability rules allow the forced transfer of an entitlement for compensation while property rules require the consent of the owner. Calabresi and Melamed (1972) famously recommended liability rules when transaction costs prevent bargaining. We ask the reverse question: Which type of rule is more conducive to bargaining? For asymmetric information—an important source of transaction costs—we show property rules to be more efficient than liability rules. Our study is motivated in part by Kaplow's and Shavell's (1995, 1996) observation that liability rules add a welfare-enhancing option, the opportunity to take when it is efficient.[1] They conjecture that, at the same time, property rules are only somewhat better at promoting efficient trade, but not enough to offset the advantage of liability rules from efficient taking. Our formal analysis puts this "a priori guess" (Kaplow and Shavell 1995, p. 224) to a test and finds it wanting. It thus restores "one of the most basic tenets of law and economics scholarship" (Ayres and Goldbart 2003, p. 123)—namely the "traditional prescription" to prefer property rules when transaction costs are low (Cooter and Ulen 2012, p. 100).

Whether the entitlement should be transferred from its current owner to a potential taker depends on the respective valuations. In our model, the owner's valuation is private information. If the parties fail to agree on a voluntary transfer, enforcing the owner's right under a property or liability rule is costly. We analyze the two polar cases of giving all bargaining power to either the uninformed taker (screening game) or the informed owner (signaling game). In the screening game, the efficiency advantage of property rules is less pronounced because the bargaining power of the taker restricts the owner's ability to benefit from her information advantage. For a small set of parameter values, liability rules even surpass property rules but this finding reverses for extensive parameter regions. The results for the signaling game are unequivocal. Vesting the informed party with full bargaining power accomplishes the first best under a property rule. Liability rules, by contrast, often entail a strictly positive probability of bargaining failure and inefficient unilateral taking.

1 Ayres and Talley (1995a, 1995b), Ayres and Goldbart (2003), and Ayres (2005) advance a more subtle argument to be discussed below.

Our results suggest that a liability rule's option to "just take" impedes the exchange of information in bargaining. Intuitively, liability rules fail to incentivize the owner to reveal his private information: If he did, the taker could capture the gains from trade by appropriating the entitlement and paying damages. Property rules force the taker to seek the owner's consent. This allows the owner to extract a share of the surplus, providing an incentive for truthful disclosure.

One crucial assumption of our model is that courts can uncover the owner's true valuation—his private information—and use it to determine the compensation award after a unilateral taking. To invert a common phrase, the owner's valuation is "unobservable but verifiable." Indeed, in an action for damages in tort or in contract, the court awards the plaintiff not some "average" amount in "this class of cases" but an estimate of the harm caused by the infringement in the particular case. While conforming to law and legal practice (see, e.g., American Law Institute 1981, §§ 347–354; 2010, §§ 26–31) our assumption deviates from parts of the earlier literature where damages under the liability rule are set at a fixed amount (Kaplow and Shavell 1996; Ayres and Talley 1995a). Treating damages as fixed may be motivated by a presumption that courts can hardly know more than the parties to the case—"verifiable" information is a subset of "observable" information. In this view, if the owner's valuation is private information, the court can determine quantum only based on the distribution, such as by always awarding the owner his mean valuation. Yet it is unpersuasive that litigation produces no more information than that of the least knowledgeable party. Litigation over damages is to a great extent about quantum. The owner (plaintiff) has the burden of proving his loss and will seek to present credible evidence, which the taker (defendant) can challenge with countervailing evidence. The effort on evidence collection, the often extensive time devoted to evaluating it and the special powers of the court—such as discovery or subpoena—all suggest that the court can have significantly more information about the owner's valuation than the taker at the original bargaining stage. Our assumption that the court learns the owner's actual valuation is meant to capture this information produced in litigation beyond the set of "observable" information at bargaining.[2]

A second important ingredient to our analysis are conflict costs. Awarding damages after a unilateral taking replaces the party's agreement with a transaction at a price determined by the court. If bargaining is costly, then so must be invoking the court as a price setter. Litigation is expensive; negotiating a settlement in the shadow of a court judgment is cheaper but not free. Insurance data for personal injury liability of Texan firms revealed a cost-to-net-payment quota of 75%, including for cases before filing suit (Hersch and Viscusi 2007). Experts have estimated the litigation expenses for a complex contract breach case over €5 million profit loss at 52% of claim value in England, 13% in Japan, and 4% in Germany (Hodges, Vogenauer, and Tulibacka 2011). While conflict costs—from settlement bargaining and litigation—can be substantial, the superiority of the property rule reflects more than a trivial tradeoff between conflict costs and allocative efficiency. The liability rule often loses on both counts or when we set conflict costs to zero. Including conflict costs nonetheless has a considerable effect in the model.

The rest of the paper is structured as follows. In part II, we discuss the related literature. Parts III and IV contain the analysis of the screening and signaling games, respectively. Part V concludes.

II. Related literature

The debate about property and liability rules originates from the seminal contribution by Calabresi and Melamed (1972). The broad and burgeoning literature often characterizes property rules as "market-encouraging" because they require mutual consent for the transfer of an entitlement, whereas liability rules are said to be

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2 A straightforward objection to our model is that the court's knowledge of the owner's private information allows the parties to devise a contractual mechanism that induces truthful disclosure by the owner already at the bargaining stage. Basically, the owner would agree to pay a large penalty if the court later found his reported valuation to exceed the true one; with a stiff enough penalty, actual litigation could be kept to a minimum. Lavie and Tabbach (2017) study a mechanism along these lines for settlement bargaining. We ignore such mechanisms because they seem to be rarely used, if at all. Explaining their absence (e.g., with risk or rent.seeking costs) requires a different paper.

"market-mimicking" (Calabresi and Melamed 1972; Haddock, McChesney, and Spiegel 1990; Craswell 1993; Cooter and Ulen 2012; see the survey by Rizzolli 2008). As has been mentioned at the outset, Kaplow and Shavell (1995, 1996) make the more far-reaching claim that liability rules are superior throughout. They frame the issue as a race between the two types of entitlement protection. The liability rule has a head start if bargaining is impossible; if transaction costs are zero, the Coase theorem leads to a tie. Kaplow's and Shavell's conjecture is that the property rule in the intermediate range of transaction costs tends never to make up for the liability rule's initial lead.

Ayres and Talley (1995a, 1995b), Ayres and Goldfarb (2003), and Ayres (2005) take this claim even further. They see the advantage of liability rules not only in efficient unilateral taking in the absence of agreement. Rather, they contend that a liability rule performs even better at reducing transaction costs by fostering the disclosure of private information. In their view, a liability rule provides the owner with an additional incentive to reveal a particularly high valuation by offering a payment to avert a potential taking. At first blush, this directly contradicts our finding. Yet Ayres, Talley, and Goldfarb make it entirely clear that their claim hinges on a preset compensation for unilateral takings. Their liability rule amounts to a call option with a fixed strike price. The information-forcing effect of liability in their analysis critically depends on courts not "tailoring" damages to the owner's specific valuation (Ayres and Talley 1995a, pp. 1065–1069). Our model builds precisely on the assumption that courts "tailor" compensation to the owner's concrete loss. The contributions of Ayres, Talley, and Goldfarb could be seen as a design prescription for optimal liability rules, whereas our approach is more in line with the law as it stands.[3] At the same time, we suspect that the law would find it difficult to adopt Ayres', Talley's, and Goldfarb's recommendation because it would require the parties to know the compensation amount at the bargaining stage.

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[3] Ayres and Talley (1995a, p. 1040) admit that "courts often attempt to tailor damages to equal the plaintiff's lost value" but claim that "there are several contexts in which the damages are sufficiently untailored—i.e., they sufficiently diverge from the plaintiff's actual valuation—to give plaintiffs an incentive to signal whether their valuation is above or below the expected court award." As examples, they point to liquidated damages in contracts and "unverifiable" damages.

Although Johnston (1995) considers only property protection, he effectively provides a similar result as Ayres, Talley, and Goldfarb. He shows that uncertainty over the allocation of an entitlement under a property rule can foster bargaining as compared to a certain allocation. Croson and Johnston (2000) confirm this prediction in an experiment. But again, their finding crucially depends on knowledge of the parties, this time about the probability of the court attributing the entitlement to either one of them.

Our paper studies negotiation under incomplete information. More specifically, the parties in our model bargain in the shadow of an impending judgment while only one of them—the owner—knows how the court will decide. This puts us close to the extensive literature on settlement bargaining (Bebchuk 1984; Reinganum and Wilde 1986; Daughety and Reinganum 1994, 1995; Schwartz and Wickelgren 2009; Farmer and Pecorino 2013; Rapoport, Daniel, and Seale 2008; Schrag 1999; Schweizer 1989; for overviews Daughety and Reinganum 2012, 2014). There is, however, a key difference: Negotiating over a settlement has the sole purpose of avoiding costly litigation. Both parties know that trade is efficient and disagree only about the distribution of gains. In our setting, bargaining is not just about saving conflict costs but also about allocating the entitlement efficiently. This sets us apart from the settlement bargaining literature. In the taxonomy of Ausubel, Cramton and Deneckere (2002), ours is a case with “no gap” between the valuation distributions of the owner and the taker. Accordingly, the literature about settlement bargaining has nothing to say about entitlement protection.

The choice between property and liability rules has a straightforward application in the law of contracts: the debate over efficient breach (Shavell 1980, 1984; Schwartz and Edlin 2003; Miceli 2004; Eisenberg 2005; Schwartz and Scott 2008). Liability protection translates into expectation damages as the only remedy for breach of contract, whereas a property rule would correspond to granting the promisee a right to specific performance (Kronman 1978). Interestingly enough, the Calabresi-Melamed framework has never gained much traction in contract law (Ayres and Goldbart 2003, p. 128), arguably because the “traditional prescription” to use property rules when bargaining is possible seemed inconsistent with the common law’s reservation towards the specific-performance remedy. In the contract literature, the possibility to negotiate around specific performance if it is inefficient (Schwartz

1979) was dismissed because it involved bargaining costs that would not arise if the promisor could unilaterally decide to breach and pay monetary damages (Kronman 1978; Shavell 1984, 2006). But as Eisenberg (2005) observes, efficient breach works less well if the promisor is uncertain about the value of performance to the promissee. Renegotiating the contract then is not a wasteful activity but a way to elicit information about the continuing efficiency of performance. Our results suggest that specific performance, by forcing the promissor to seek the promissee's consent for non-performance, encourages information exchange between the parties.

III. Screening game

1. Model

Our model concerns the "holder" or "owner" of an entitlement ("he") and a potential "taker" ("she"). The owner's valuation o of the entitlement is private information; the taker's valuation t is common knowledge. Both valuations are drawn independently from a uniform distribution over the interval $[0, H]$ with $H > 0$. In the screening model, the taker makes a take-it-or-leave-it offer to buy the entitlement at price x. If the owner accepts, he receives a payoff $\Pi_O = x$; the taker's payoff is $\Pi_T = t - x$. If the owner rejects, continuation of the game depends on the available remedy.

Under a *liability rule*, the taker can choose to take unilaterally. The owner can enforce a claim for monetary damages. The court observes o and orders the taker to pay o as expectation damages to the owner. However, litigation and settlement bargaining impose expected conflict costs ϕ on each party. Unilateral taking thus results in payoffs $\Pi_O = o - \phi$ for the owner and $\Pi_T = t - o - \phi$ for the taker. We assume that the owner always seeks damages if the taker infringes his right, incurring cost ϕ.[4]

[4] If $o < \phi$, the holder may derive additional utility from taking the taker to account, or she might seek to preserve a reputation for defending her rights. We could as well assume $o \in [L, H]$ with $L > \phi$ but wanted to save notation.

Still under a liability rule, if the taker abstains from appropriating the entitlement, no costly conflict seems to arise. Yet respecting a right may not be trivial. The parties can disagree over the scope and content of the owner's right. For instance, a dispute can arise over whether a patent extends to a particular technological process or—in a contractual setting—whether the promisor's performance meets the contractual specification. To account for such disputes, we include conflict cost ψ for each party if there is no agreement and the taker chooses to respect the entitlement. Hence, the payoffs are $\Pi_O = o - \psi$ and $\Pi_T = -\psi$.

Figure 1 shows the game tree under a liability rule without nature's choice of o and t from $[0, H]$.

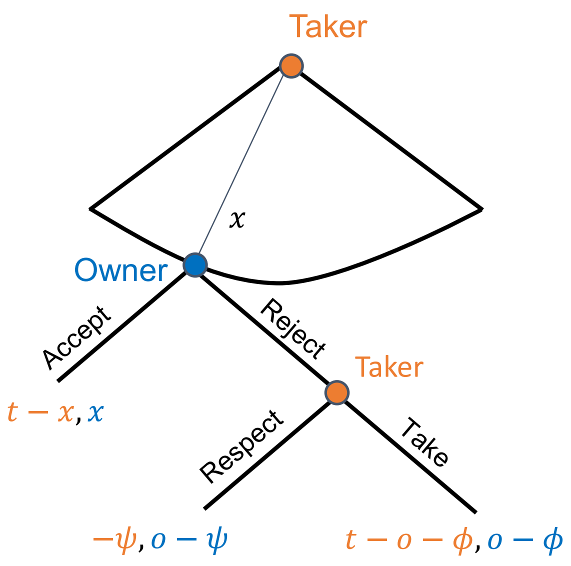

Figure 1: Screening game with liability rule

With a *property rule* as the remedy, if the parties fail to reach agreement the owner enforces his right and prevents the taker from infringing. As under the liability rule, forcing the taker to respect the entitlement imposes conflict cost ψ on both parties. This reflects the cost of having the court grant an injunction against the taker or determining the precise scope of the entitlement by mutual agreement. Remember that $\psi < \phi$ so that the holder never prefers to collect damages instead of enforcing his entitlement. As a result, payoffs amount to $\Pi_O = o - \psi$ and $\Pi_T = -\psi$. The game tree in Figure 2 depicts the simpler situation under a property rule.

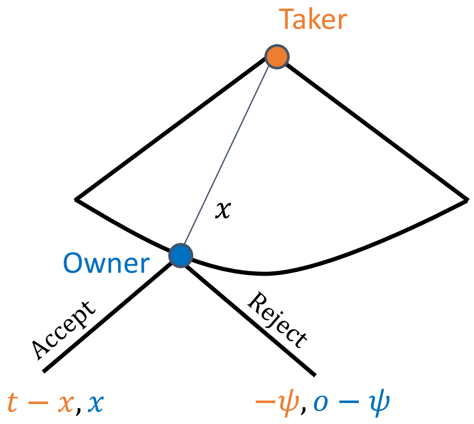

Figure 2: Screening game with a property rule

Before examining the equilibria and welfare consequences under the property and liability rules, we fix the first best as a reference. Neither type of entitlement protection allows the parties to save conflict costs ψ if they choose to maintain the initial entitlement. Therefore, we include conflict costs in the first best. Under first-best behavior, the taker never infringes the entitlement even under a liability rule because a voluntary transfer is always cheaper than incurring conflict costs ϕ. But conflict costs ψ for enforcing or monitoring the entitlement when the original allocation is preserved. The total first-best payoff for both parties then obviously is

$$\Pi_{FB} = \begin{cases} t & o < t + 2\psi \\ o - 2\psi & o \geq t + 2\psi \end{cases}$$

The expected value in o then is

$$\mathrm{E}_o(\Pi_{FB}) = \min\left(\frac{t + 2\psi}{H}, 1\right) t + \left(1 - \min\left(\frac{t + 2\psi}{H}, 1\right)\right)\left(\frac{t + 2\psi + H}{2} - 2\psi\right)$$

That ψ enters the first best highlights a peculiar assumption, namely that conflict costs ψ arise only for enforcing the original allocation but not the one resulting from a voluntary transfer. One can justify this asymmetry with the idea that in an agreement the parties can devise a tailored, cheaper property protection for the new entitlement. In any event, imposing conflict costs ψ universally would mean that they even out and no longer affect equilibria and optimal choices. The model can accommodate this case by setting $\psi = 0$.

2. Equilibria

We start by characterizing the solution of the screening game under the property rule. The taker's problem corresponds to that of a price-setting monopsonist with no ability to price discriminate. In making an offer, she trades off the chance to strike a deal with a higher-valuation owner against overpaying a low-valuation owner. The following Proposition 1 states the resulting equilibrium. Proofs are relegated to the appendix.

Proposition 1. Equilibrium of the screening game under the property rule

(I) For low $H \leq 2\psi$:

If $t \leq 2H - 2\psi$, the taker offers $x = \frac{t}{2}$ and owners with $o \leq x + \psi$ accept.

If $t > 2H - 2\psi$, the seller offers $t = H - \psi$ and all owners accept.

(II) For high $H > 2\psi$, the taker offers $x = \frac{t}{2}$ and owners with $o \leq x + \psi$ accept.

The taker knows that without an agreement the owner will prevent her from infringing the entitlement, imposing the additional cost ψ of forced compliance on both parties. Her payoff-maximizing offer is $x = \frac{t}{2}$, implying that efficient agreements with owners in the range $\frac{t}{2} + \psi < o \leq t + \psi$ are foregone. Only if conflict costs are very large (that is, for low $H \leq 2\psi$), there is an additional possibility: A taker's valuation could be so high that she indiscriminately prefers to buy out all owners.

The following Proposition 2 contains the equilibrium under liability protection:

Proposition 2. Equilibrium of the screening game under the liability rule

(I) For low $H \leq H^{I} = \frac{11}{8}\phi + \frac{5}{8}\psi - \frac{1}{8}\sqrt{25\phi^2 - 18\phi\psi - 7\psi^2}$:

Takers with $t \leq \bar{t}^{N} = 2H - 2\psi$ offer $x = \frac{t}{2}$ and owners with $o \leq \frac{t}{2} + \psi$ accept; if rejected, the taker respects the entitlement.

Takers with $\bar{t}^{N} < t \leq t^{\bar{T}\bar{N}} = \frac{H}{2} + 2\phi - \psi - \frac{1}{2}\sqrt{H^2 - 2H\phi + 4\phi^2 - 2H\psi}$ offer $x = H - \psi$ and all owners accept.

Takers with $t > t^{\bar{T}\bar{N}}$ offer $x = 2t - H - 3\phi + 2\psi$ and owners with $o \leq 2t - H - 2\phi + 2\psi$ accept; if rejected, the taker infringes the entitlement.

(II) For intermediate H with $H^{I} < H \leq H^{II} = 4\phi - 2\psi + 4\sqrt{2}\sqrt{\phi^2 - \phi\psi}$:

Takers with $t \leq t^{\bar{T}\dot{N}} = \frac{2}{9}\left(3H + 8\phi - 5\psi - \sqrt{(\phi - \psi)(3H + 10\phi + 2\psi)}\right)$ offer $x = \frac{t}{2}$ and owners with $o \leq \frac{t}{2} + \psi$ accept; if rejected, the taker respects the entitlement.

Takers with $t^{\bar{T}\dot{N}} < t \leq t^{T} = \frac{H}{2} + 2\phi - \psi$ offer $x = 2t - H - 3\phi + 2\psi$ and owners with $o \leq 2t - H - 2\phi + 2\psi$ accept; if rejected, the taker infringes the entitlement.

Takers with $t > t^{T}$ offer $x = \phi$ and owners with $o \leq 2\phi$ accept; if rejected, the taker infringes the entitlement.

(III) For high $H > H^{II}$:

Takers with $t \leq t^{\dot{T}\dot{N}} = 2H - 2\psi - \sqrt{2}\sqrt{H^2 - 2H\phi + 4\phi^2 - 2H\psi}$ offer $x = \frac{t}{2}$ and owners with $o \leq \frac{t}{2} + \psi$ accept; if rejected, the taker respects the entitlement.

Takers with $t > t^{\dot{T}\dot{N}}$ offer $x = \phi$ and owners with $o \leq 2\phi$ accept; if rejected, the taker infringes the entitlement.

The different value ranges of H capture the level of conflict costs relative to the variance of the taker's and the owner's valuation. "Low H" thus means "high conflict costs." But the relative size of ψ and ϕ, the two types of conflict costs, also affects the case distinction. Figure 3 depicts this relationship, where the proposition's case (I) corresponds to the black area, case (II) to the dark gray area, and case (III) to the two light gray areas to the left. Because we have normalized the distributions of t and o to an interval from zero to H, one should not read H as the maximum value of the entitlement. Rather, it captures the variance in valuations. Empirical estimates of litigation costs as a percentage of litigated claims do not translate into $\frac{\phi}{H}$ or $\frac{\psi}{H}$.

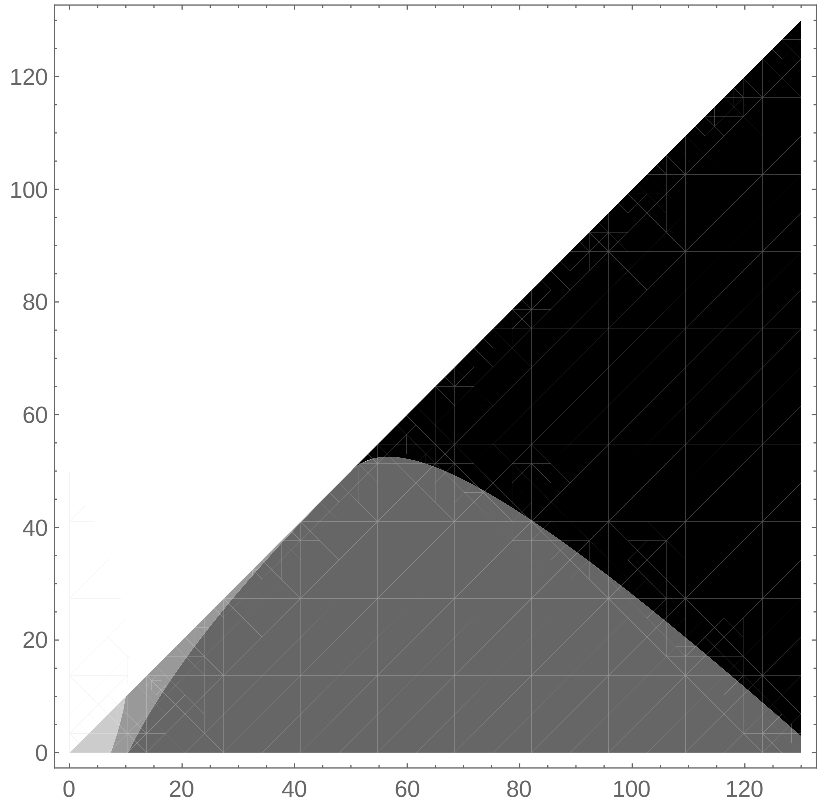

Figure 3: The areas indicate different ranges of conflict costs ϕ (horizontal axis) and ψ (vertical axis) for $H = 100$. The black area represents Proposition 2 (I) with high conflict costs ($100 \leq H^I$), the dark gray area Proposition 2 (II) with intermediate conflict costs ($H^I < 100 \leq H^{II}$), and the two lighter gray areas Proposition 2 (III) with low conflict costs ($100 > H^{II}$).

Figure 4 illustrates the equilibria from Proposition 2. One main insight is that outcomes differ between property and liability protection only for higher taker valuations. Depending on conflict costs, the taker is committed to respecting the entitlement for a broad range of valuations when her offer is rejected. The reason is that the expected damages plus conflict costs ϕ exceed the benefits from taking plus the savings in conflict costs ψ. In this range, the seller seeks to acquire the entitlement, mostly by offering the optimal monopsonist price $x = \frac{t}{2}$, just as she would under a property rule.

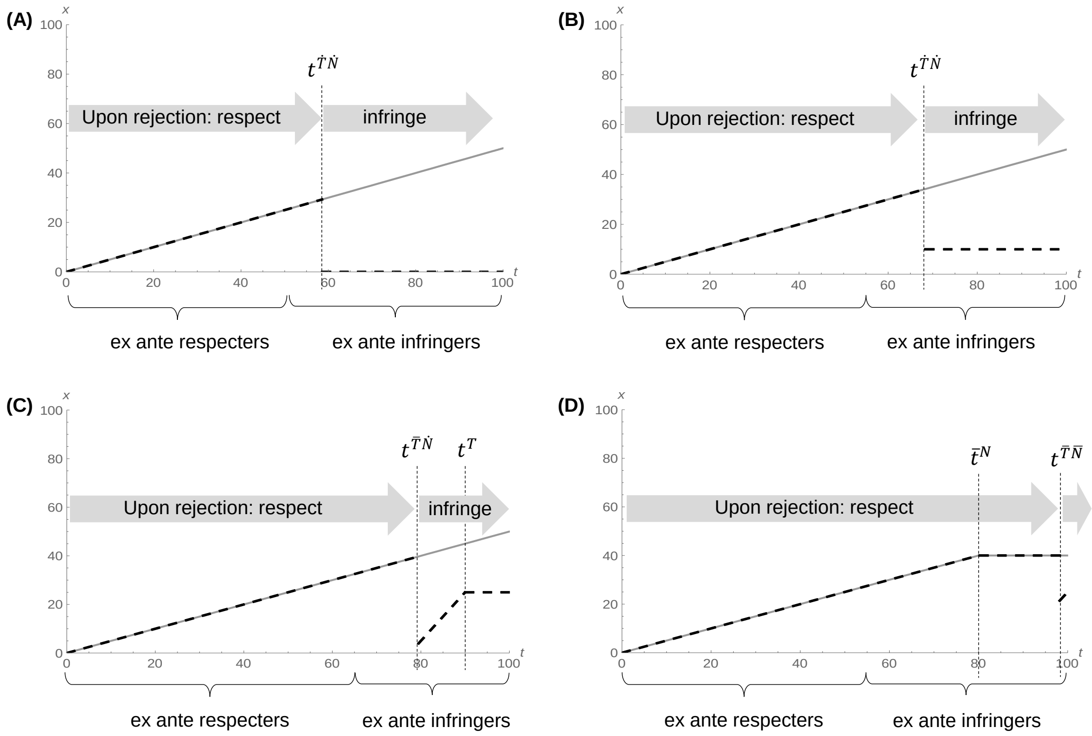

Figure 4: Equilibrium taker offers x as a function of taker's valuation t with $H = 100$. The gray lines depict taker offers under the property rule, the dashed lines under the liability rule. "Ex ante respecters" are takers who would have respected the entitlement under a liabolity rule in the absence of bargaining. Pane (A) and pane (B) show Proposition 2 (III) with low conflict costs ($\phi, \psi = 0$ and $\phi = 10, \psi = 5$, respectively). Pane (C) reflects Proposition 2 (II) with intermediate conflict costs ($\phi = 25, \psi = 10$) and pane (D) Proposition 2 (I) with high conflict cost ($\phi = 65, \psi = 60$).

The opportunity to buy out lower-valuation owners also raises the taker's threshold for infringing the entitlement after rejection because the remaining owners have a larger valuation o on average, which they can claim as damages if their right is taken. Bargaining produces information for the taker even if it breaks down, as the following remark states.

Remark 1

> Under liability protection, bargaining leads more takers to respect the entitlement: All three threshold values $t^{\bar{T}\bar{N}}$, $t^{\bar{T}\dot{N}}$, and $t^{\dot{T}\dot{N}}$ for the taker respecting the entitlement after rejection exceed the taker valuation threshold $\frac{H}{2} + \phi - \psi$ above which the taker would infringe in the absence of bargaining.

For valuations greater than $t^{\bar{T}\bar{N}}$, $t^{\bar{T}\dot{N}}$, and $t^{\dot{T}\dot{N}}$, respectively, takers are committed to appropriate the entitlement if no agreement is reached. They continue to screen, but only for owners with valuations low enough to warrant buying them out and avoiding conflict cost ϕ from an unconsented taking. Usually, takers offer $x = \phi$ to screen out such owners (Figure 4, pane (B)), which owners with $o \leq 2\phi$ accept. Yet sometimes an additional constraint arises: For takers with $t \leq t^T = \frac{H}{2} + 2\phi$, offering the full ϕ would raise the expected liability towards rejecting owners by so much that it would become optimal for the taker to respect, rather than infringe, upon rejection. This would make owners less willing to accept. Takers therefore restrict their offers to preserve the owners' pure belief in her commitment to take (Figure 3, pane (C) for $t^{\bar{T}\dot{N}} < t \leq t^T$, and pane (D) for $t > t^{\bar{T}\bar{N}}$).

3. Welfare

Figure 4 suggests that bargaining succeeds more often with property than with liability protection. Except for very large conflict costs ψ, takers always offer $x = \frac{t}{2}$ under a property rule. Under a liability rule, while lower-valuation takers make the same offer $x = \frac{t}{2}$, takers with high valuations make rather unattractive offers.

The welfare consequences also depend on the response to bargaining failure. In this regard, a property rule has the disadvantage of preventing takers from appropriating the entitlement even if their valuation is very high. The following proposition shows that the benefits of property protection usually outweigh this shortcoming.

Proposition 3. Welfare comparison of equilibria under the property and liability rules

(I) For low $H \leq H^{I} = \frac{11}{8}\phi + \frac{5}{8}\psi - \frac{1}{8}\sqrt{25\phi^2 - 18\phi\psi - 7\psi^2}$:

For $t \leq t^{\bar{T}\bar{N}} = \frac{H}{2} + 2\phi - \psi - \frac{1}{2}\sqrt{H^2 - 2H\phi + 4\phi^2 - 2H\psi}$, both rules are equally efficient;

for $t > t^{\bar{T}\bar{N}}$, the property rule is more efficient.

(II) For intermediate H with $H^{I} < H \leq H^{II} = 4\phi - 2\psi + 4\sqrt{2}\sqrt{\phi^2 - \phi\psi}$:

For $t \leq t^{\bar{T}\dot{N}} = \frac{2}{9}\left(3H + 8\phi - 5\psi - \sqrt{(\phi - \psi)(3H + 10\phi + 2\psi)}\right)$, both rules are equally efficient;

for $t > t^{\bar{T}\dot{N}}$, the property rule is more efficient

(III) For high H with $H^{II} < H \leq H^{III} = 8\phi - 2\psi + 4\sqrt{2\phi^2 - 2\phi\psi + \psi^2}$:

For $t \leq t^{\dot{T}\dot{N}} = 2H - 2\psi - \sqrt{2}\sqrt{H^2 - 2H\phi + 4\phi^2 - 2H\psi}$, both rules are equally efficient;

for $t > t^{\dot{T}\dot{N}}$, the property rule is more efficient.

(IV) For high $H > H^{III}$:

For $t \leq t^{\dot{T}\dot{N}} = 2H - 2\psi - \sqrt{2}\sqrt{H^2 - 2H\phi + 4\phi^2 - 2H\psi}$, both rules are equally efficient;

for $t^{\dot{T}\dot{N}} < t \leq t_{PRLR} = \frac{4}{3}H - 2\psi - \frac{2}{3}\sqrt{H^2 - 12H\phi + 24\phi^2}$, the property rule is more efficient;

for $t > \frac{4}{3}H - 2\psi - \frac{2}{3}\sqrt{H^2 - 12H\phi + 24\phi^2}$, the liability rule is more efficient.

The liability rule prevails only in case (IV) of Proposition 3, for very low conflict costs ($H > H^{III}$), and even there only for certain taker valuations ($t > t_{PRLR}$). The single lightmost gray area to the very left in Figure 3 shows the region of conflict costs where the liability rule can be superior. For higher conflict costs that fail to satisfy $H > H^{III}$, the two types of entitlement protection are either equivalent or the property rule is more efficient.

Figure 5 illustrates the findings from Proposition 3. The curves represent the parties' total expected payoffs as a function of the taker's valuation t for the different cases of Proposition 3. As a reference, the upmost thin line shows the first-best total expected payoff from subsection III.1 where owner and taker agree whenever it is efficient in light of valuations and conflict costs ψ. Because of ψ, the first-best payoff approaches zero for low t as in pane (D) of Figure 5.

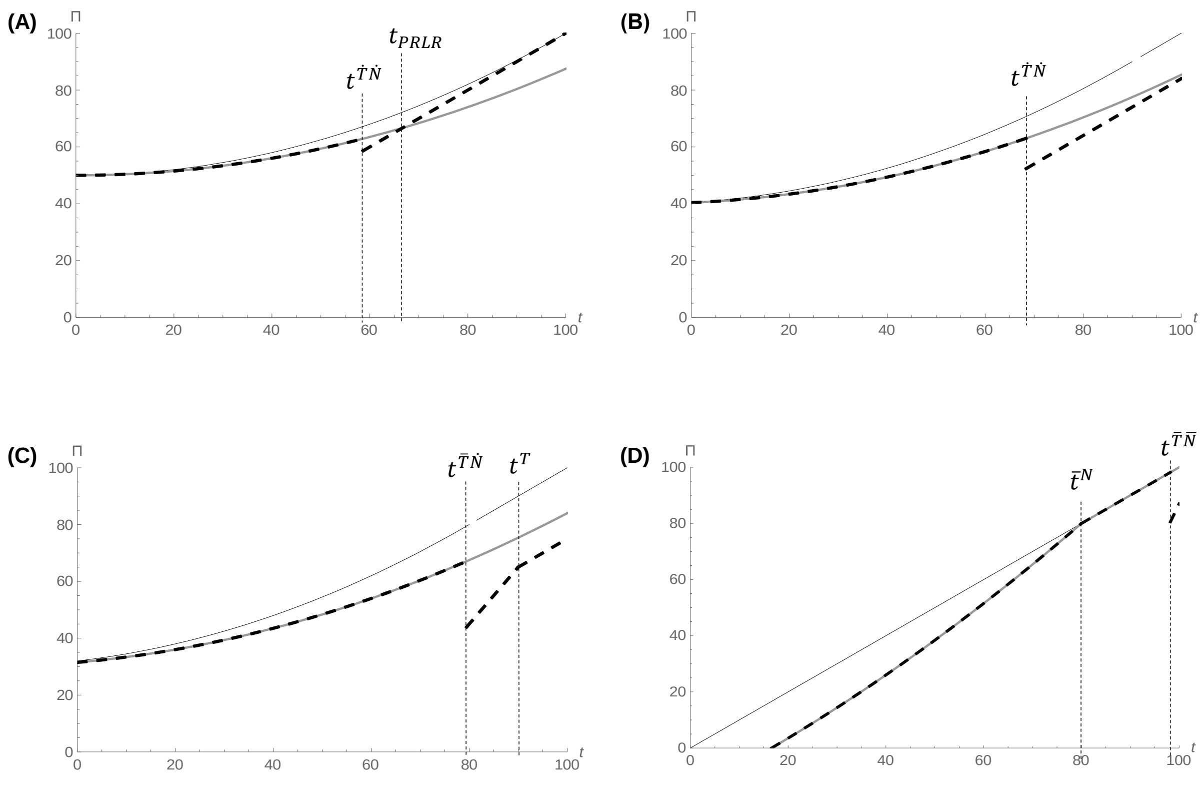

Figure 5: Total payoffs Π as a function of taker's valuation t with $H = 100$. The upper thin lines represent the first-best total payoffs, the gray and dashed lines total payoffs under the property rule and the liability rule, respectively. Pane (A) shows Proposition 3 (IV) with very low conflict costs ($\phi, \psi = 0$) and Pane (B) contains an example of Proposition 3 (III) with low conflict costs ($\phi = 10, \psi = 5$). Pane (C) reflects Proposition 3 (II) with intermediate conflict costs ($\phi = 25, \psi = 10$) and pane (D) Proposition 3 (I) with high conflict cost ($\phi = 65, \psi = 60$).

1. Model

In the signaling model, it is for the owner to make a demand x. If the taker accepts, the owner's payoff is $\Pi_O = x$ whereas the taker receives $\Pi_T = t - x$. If the demand is rejected, continuation again hinges on the available remedy.

With a *liability rule*, the taker can choose to respect the entitlement, implying a payoff $\Pi_T = -\psi$ for her and a payoff to the owner of $\Pi_O = o - \psi$. If the taker appropriates the entitlement, she gets $\Pi_T = t - o - \phi$ and the owner $\Pi_O = o - \phi$. As in the screening model, we assume $\psi < \phi$ and that the owner always sues when his right is usurped. Figure 6 shows the game tree without nature's move choosing $o, t \in [0, H]$.

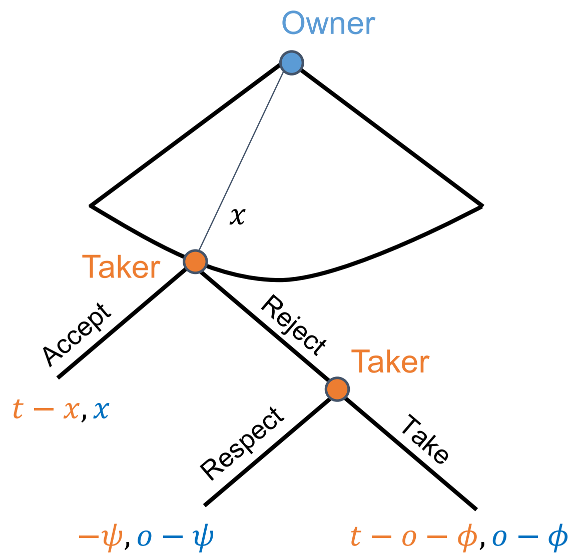

Figure 6: Signaling game with liability rule

Under a *property rule*, the taker has to respect the entitlement if no agreement is made. Payoffs then are $\Pi_O = o - \psi$ and $\Pi_T = -\psi$. The signaling game is summarized in Figure 7.

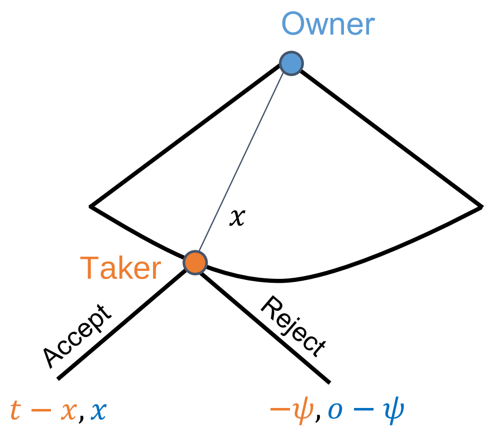

Figure 7: Signaling game with property rule

The first-best payoffs under the signaling game correspond to those under the screening game in subsection III.1.

2. Equilibria

In the signaling game, the owner combines complete information and all the bargaining power. A property rule permits him to capture all available surplus, leading to full separation:

Proposition 4. Equilibrium of the signaling game under the property rule

> An owner with $o \leq t + 2\psi$ demands $x = t + \psi$, which the taker accepts. If the owner's valuation is higher, $o > t + 2\psi$, he makes an unacceptable offer $x > t + \psi$ that the taker rejects.

The reasoning behind Proposition 4 is straightforward: The taker accepts all offers $x \leq t + \psi$. The owner demands the highest acceptable price $x = t + \psi$ if this makes him better off than the payoff $o - \psi$ from keeping the entitlement.

The equilibrium with expectation damages is more complicated.

Proposition 5. Equilibrium of the signaling game under the liability rule

Let $\bar{o} = 2\phi \ln \frac{2\phi}{\phi - \psi}$ and $\bar{\bar{o}} = 2t + 2\psi - 2\phi - H$.

(I) For low taker valuations $t \leq 2\phi$, there is a semi-separating equilibrium in pure strategies:

Owners with $o \leq t + 2\psi$ demand $x = t + \psi$, which takers accept.

Owners with $o > t + 2\psi$ demand $x > t + \psi$; takers reject and respect the entitlement.

(II) For lower intermediate taker valuations with $2\phi < t \leq \bar{o} + \phi - \psi$, there is a semi-separating equilibrium with mixed taker strategies:

Owners with $o \leq t + \psi - \phi$ demand $x = o + \phi$, which takers accepts with probability $p(x) = e^{\frac{\phi - x}{2\phi}}$; otherwise, they reject and infringe the entitlement.

Owners with $o > t + \psi - \phi$ demand $x > t + \psi$; takers reject and respect the entitlement.

(III) For higher intermediate taker valuations with $\bar{o} + \phi - \psi \; < t \leq \frac{H + \bar{o}}{2} + \phi - \psi$, there is a semi-separating equilibrium with mixed taker strategies:

Owners with $o \leq \bar{o}$ demand $x = o + \phi$, which takers accept with probability $p(x) = e^{\frac{\phi - x}{2\phi}}$; otherwise, they reject and infringe the entitlement.

Owners with $o > \bar{o}$ demand $x > t + \psi$; takers reject and respect the entitlement.

(IV) For high taker valuations with $\frac{H + \bar{o}}{2} + \phi - \psi < t \leq H$, there is a semi-separating equilibrium with mixed taker strategies:

Owners with $o \leq \bar{\bar{o}}$ demand $x = o + \phi$, which takers accept with probability $p(x) = e^{\frac{\phi - x}{2\phi}}$; otherwise, they reject and infringe the entitlement.

Owners with $o > \bar{\bar{o}}$ demand $x > t + \psi$; takers reject and respect the entitlement with probability $\pi = \frac{2\phi}{\phi - \psi} e^{-\frac{2t + 2\psi - 2\phi - H}{2\phi}}$; otherwise they reject and infringe the entitlement.

For a small range of low taker valuations $(t \leq 2\phi)$, we find a pure strategy equilibrium that equals the one under the property rule in Proposition 4. This equilibrium is driven by conflict costs, which prevent the taker from seizing the entitlement. With a higher taker valuation, the equilibria deviate from the one under the property rule. The owner no longer can claim all the surplus from trade due to the taker's option to appropriate the entitlement unilaterally. The resulting equilibria involve mixed strategies by the taker. All three equilibria share the common feature that owners separate in two groups: The lower-valuation owners make the fully revealing demand $x = o + \phi$; takers randomize between accepting and rejecting followed by taking ("reject-take" for short). Higher-valuation owners make demands that no taker accepts. The equilibria differ in cutoffs between the two owner groups as well as in how takers respond to inacceptable demands.

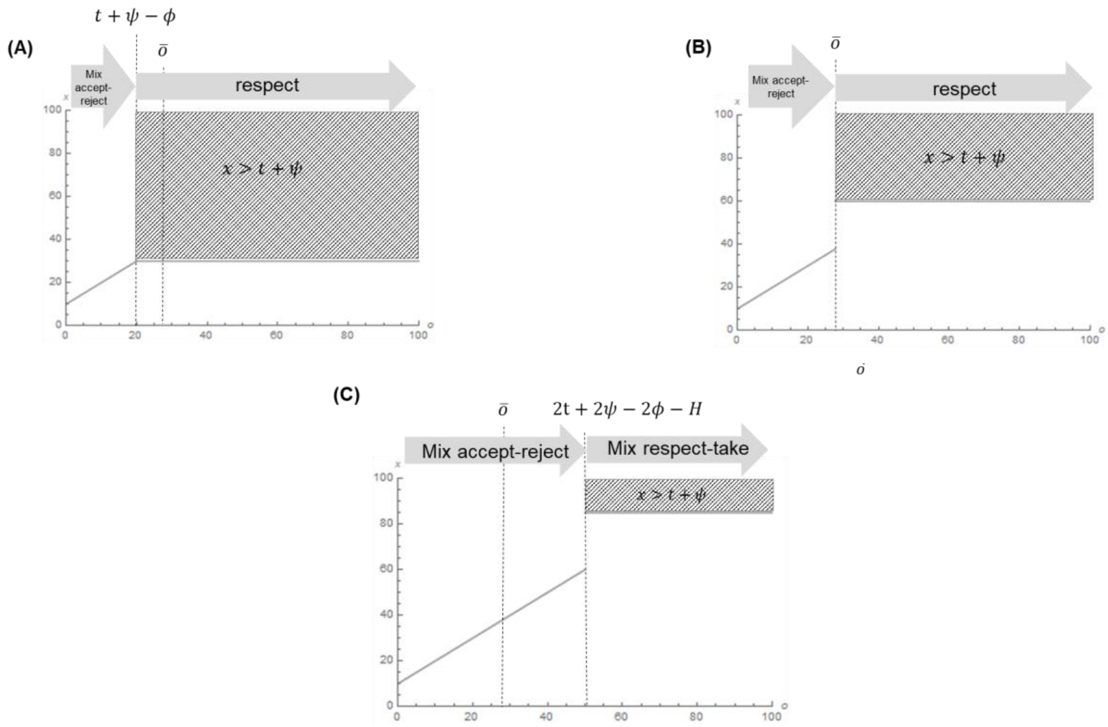

Figure 8: Equilibrium owner demands x as a function of owner's valuation o with $H = 100,\ \phi = 10$, and $\psi = 5$ under the liability rule. The hatched area represents inacceptable demands $x > t + \psi$. Pane (A) exemplifies Proposition 5 (II) with $t = 25$, pane (B) reflects Proposition 5 (III) with $t = 55$, and pane (C) Proposition 5 (IV) with $t = 80$.

Figure 8 summarizes the interesting cases (II)–(IV) from Proposition 5. Equilibrium of the signaling game under the liability rule

To grasp the intuition behind the equilibrium, start by considering pane (A) for "lower intermediate" taker valuations, reflecting Proposition 5 (II). Takers reject demands $x > t + \psi$ and subsequently respect the entitlement because owners making such high demands are from the higher-valuation group. To induce owners from the lower-valuation group with $o \leq t + \psi - \phi$ to make revealing demands $x = o + \phi$, takers randomize between accept and reject-take. The acceptance probability must prevent the owner from mimicking, firstly, a higher type within the low-valuation (separating) group and, secondly, a type from the high-valuation (pooling) group. The first constraint requires that a higher demand is associated with a lower probability of acceptance, as $p(x)$ in Proposition 5 provides. For the second constraint to be met, the acceptance probability cannot fall below a certain threshold. The cutoff $\bar{o} = 2\phi \ln \frac{2\phi}{\phi - \psi}$ reflects this limitation—it is the highest owner valuation for which randomization with acceptance probability $p(x)$ can induce separating demands, given the second constraint.

The threshold $\bar{o}$ is not yet binding in case (II) of Proposition 5 it is preempted by another constraint, namely that for the taker to randomize between accept and reject-breach, the demand $x = o + \phi$ must generate a higher payoff than rejecting, followed by respecting the entitlement ("reject-respect" for short). The latter constraint yields the cutoff $t + \psi - \phi$ in Proposition 5 (II). In case (III), $\bar{o}$ becomes binding. Owners with higher valuations no longer separate. Instead, they make inacceptable demands. Interestingly, this implies that certain demands are not made at all in equilibrium, as Figure 8 pane (B) shows.[5] Because taker valuations in Proposition 5 (III) are still at an "intermediate" level, takers respond with reject-respect.

With the high taker valuations of case (IV), a pure reject-respect response to an inacceptable demand as in case (III) is no longer in equilibrium: If it still were that all owners above $\bar{o}$ made an inacceptable demand, the taker would respond reject-take rather than respect-respect. But such a pure response would make it profitable for owners with valuations in the lower range of $]\bar{o}, H]$ to differentiate themselves by off-equilibrium demands between separating and inacceptable ones, that is, $\bar{o} + \phi < x \leq \frac{H+\bar{o}}{2} + \phi$. Takers would rather accept such a demand ($\Pi_T = t - x$) than reject-take (with $\Pi_T = t - \frac{H+\bar{o}}{2} - \phi$, given equilibrium play). The equilibrium with a pure reject-taker response would unravel.

There is, however, a viable equilibrium with a mixed taker strategy between reject-respect and reject-take as stated in Proposition 5 (IV). Randomizing between respecting and infringing the entitlement after rejection worsens the owner's payoff from making an inacceptable demand and loosens the constraint for separating demands; this allows Proposition 5 (IV) to extend the range of separating demands beyond $\bar{o}$. The randomization probability π indirectly determines the new owner valuation threshold $\bar{\bar{o}}$ for separating demands. $\bar{\bar{o}}$ and hence π need to ensure that the

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[5] In the proof, we use the "intuitive" and the "divine" criterion to determine whether the equilibrium is robust to off-equilibrium demands.

taker is indifferent between respecting and infringing the entitlement when she confronts an inacceptable demand.

3. Welfare

In the signaling model, the property rule is unambiguously superior from a welfare perspective. It always leads to efficient agreements whereas with a liability rule, the parties' ability to conclude efficient agreements depends on the taker's valuation. For higher valuations, there is a strictly positive probability under the liability rule that the parties forego efficient trading opportunities. The following Proposition 6 states this result.

Proposition 6. Welfare comparison of equilibria under the property and liability rules

(I) For low taker valuations $t \leq 2\phi$, property and liability rules are equally efficient.

(II) For high taker valuations $t > 2\phi$, property rules are more efficient.

Figure 9 illustrates the welfare loss from a liability rule. In a setting of low taker's valuations ($t \leq 2\phi$), the parties behave equally under both rules, implying the same welfare outcome. With $2\phi < t \leq \frac{H+\bar{o}}{2} + \phi - \psi$ (Proposition 5 (II and III)), the liability rule produces two types of welfare losses: The taker does not always accept low demands but rejects with a certain probability and then infringes the entitlement; this causes unneeded conflict costs. In addition, the taker respects the entitlement of owners with $t + \psi - \phi < o \leq t + 2\psi$ although a transfer would be efficient.

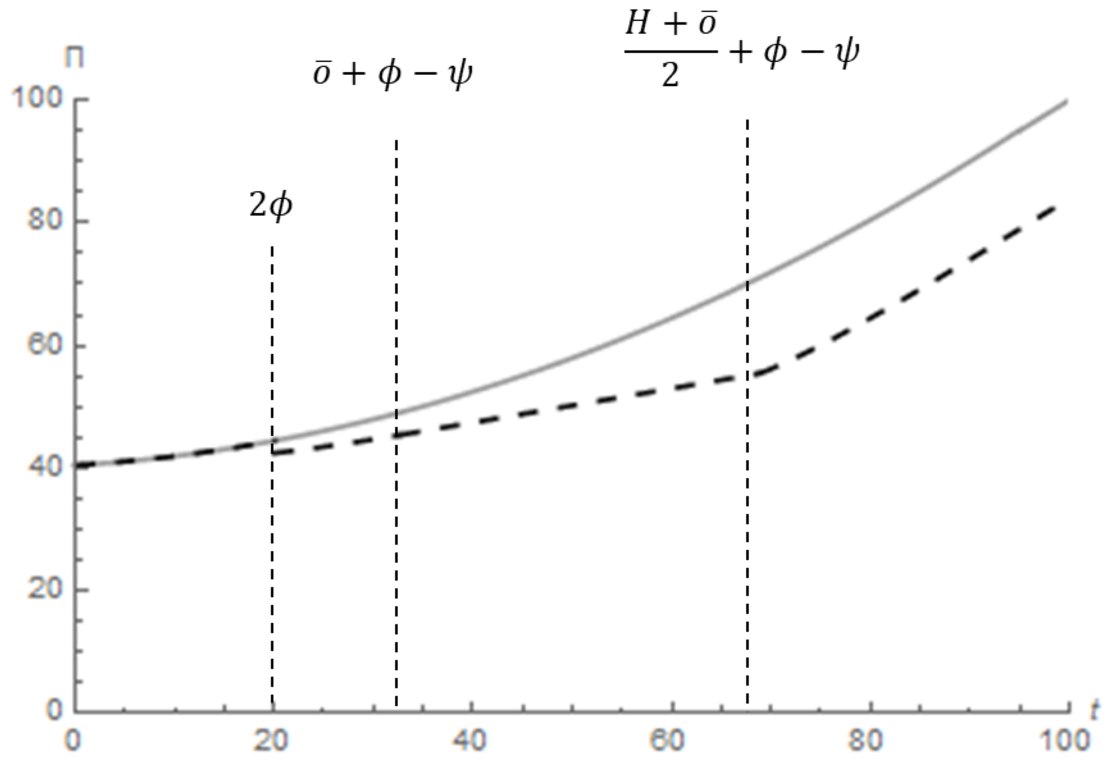

Figure 9: Total payoffs Π as a function of taker's valuation t with $H = 100$, $\psi = 5$, and $\phi = 10$. The gray line represents the total payoff under the property rule, which equals the first best, the dashed line depicts the total payoff under the liability rule.

The first type of welfare loss increases with taker valuations because the probability with which she rejects demands followed by taking rises when facing higher valuing owners where agreements would be possible.

In addition, the welfare gap increases because the greater the taker's valuation, the more owners with whom agreements would be efficient make demands the taker never accepts and thereby induces her to respect the entitlement.

For high taker valuations $\frac{H+\bar{o}}{2}+\phi-\psi<t\leq H$ (Proposition 5 (IV)), the difference in welfare stems less from the taker inefficiently respecting the entitlement. Due to her mixed response between reject-respect and reject-take, more owners make a revealing demand instead of mimicking high-valuation owners. However, the mixed strategy involves inefficient takings when she gets a high demand and the owner having a higher valuation.

V. Discussion

[To be added]

VI. Conclusion

Incomplete information increases transaction costs. This makes property rules less attractive because they demand the consent of the owner to transfer an entitlement. This efficiency advantage of liability rules shows in the screening model of the present paper. But the screening and signaling models highlight a competing effect: Incomplete information also raises the costs of forced transactions under a liability rule, both because of conflict over monetary compensation and the possibility of inefficient taking. Each of these two components can suffice for property rules to win the race, but taken together, the defeat of liability rules is all but inevitable.

Given the general character of Calabresi's and Melamed's framework, this result has important implications in a wide range of settings. The longstanding debate over contract remedies has already been mentioned. Of course, in each field other considerations will bear on the choice of entitlement protection. An example is how the distributional consequences of ex post bargaining under the different rules affect ex ante investment incentives. Other caveats relate to limitations in the assumptions: We have examined only the extreme cases of giving all bargaining power to either the owner or the taker. Also, real-world situations often involve information asymmetries about the valuation not only of the owner but also that of the taker. Exploring these complications is left to future research.

VII. Proofs

1. Screening model

a) Proposition 1. Equilibrium of the screening game under the property rule

The owner always accepts an offer $x \geq H - \psi$ and rejects all offers $x < -\psi$. For intermediate or "screening" offers $x \in [-\psi, H - \psi[$, the owner accepts with probability $\frac{x+\psi}{H}$; the expected payoff for the taker is $\Pi_T = -\psi + \frac{x+\psi}{H}(t - x + \psi)$. As a function of x, this payoff has a maximum at $x = \frac{t}{2}$. We denote the corresponding payoff $\dot{\Pi}_{T\ PR} = \frac{\left(\frac{t}{2}+\psi\right)^2}{H} - \psi$. The taker prefers this payoff over the one from an offer $x < -\psi$, which is always rejected and yields $\Pi_T = -\psi$. He strictly prefers the optimal screening offer over the high offer $x = H - \psi$ that any owner accepts if

$$-\psi + \frac{\left(\frac{t}{2} + \psi\right)^2}{H} > t - H + \psi$$

$$0 > (t + 2\psi)H - H^2 - \left(\frac{t}{2} + \psi\right)^2$$

The right hand side reaches its maximum for $H = \frac{t}{2} + \psi$. Plugging this in gives us

$$0 > (t + 2\psi)\left(\frac{t}{2} + \psi\right) - 2\left(\frac{t}{2} + \psi\right)^2$$

which never holds. It follows that the taker prefers the screening offer $x = \frac{t}{2}$. Yet to be a "screening" offer, it also has to remain below the upper limit $o - \psi$ at which all owners accept: $\frac{t}{2} < H - \psi \Leftrightarrow t < 2H - 2\psi$. Given the assumption $t \leq H$, the latter condition can only be violated if $2\psi > H$, which yields the case distinction of Proposition 1. If $2\psi > H$, the taker's payoff is $\overline{\Pi}_{T\ PR} = t - H + \psi$.

b) Proposition 2. Equilibrium of the screening game under the liability rule

(1) The second and third stage: owner's acceptance and taker's seizure decision

We start by characterizing the equilibria of the subgame after the taker's offer: the owner's decision to accept or reject an offer, and the taker's decision to take the entitlement or abstain from doing so after she has been rejected.

All-accept equilibrium, $x \geq H - \psi$**.** Very high offers $x \geq H - \psi$ are always accepted. We denote the taker's payoff in this case $\Pi_T^A = t - x$.

Never-take equilibrium, $x \in [2t - H - 2\phi + \psi, H - \psi[$. For lower offers, not all owner types accept. In the "never take" subgame equilibrium, if the owner rejects the taker always refrains from taking. Knowing this, the owner accepts if $x \geq o - \psi$. For the taker to in fact respect the entitlement, it has to be that $t - \frac{\max(x+\psi,0)+H}{2} - \phi \leq -\psi$. Thus, for offers $x < -\psi$, the taker's valuation has to satisfy $t \leq \frac{H}{2} + \phi - \psi$. These are takers that would not seize the entitlement in the absence of a negotiation; we refer to them as "ex ante respecters". But because no owner—knowing that he faces an "ex ante respecter"—accepts such offers, we relegate the case of $x < -\psi$ to the all-reject equilibrium below and consider here only higher offers. The never-take equilibrium then requires that $x \geq 2t - H - 2\phi + \psi$. The taker's corresponding payoff—omitting the LR subscript—is

$$\Pi_T^N = \frac{x + \psi}{H}(t - x) - \left(1 - \frac{x + \psi}{H}\right)\psi$$

$$\Pi_T^N = \frac{x + \psi}{H}(t - x + \psi) - \psi$$

Mixed-strategy equilibrium, $x \in]2t - H - 3\phi + 2\psi, 2t - H - 2\phi + \psi[$**.** For offers below the threshold $2t - H - 2\phi + \psi$, we again exclude offers that all owner types reject (i.e., $x < -\phi$ and, for "ex ante respecters", $x < -\psi$; see the discussion of the all-reject equilibrium below). Here, we consider only offers that at least some owner types accept. Suppose that, in response to rejection, the taker always appropriates the entitlement. Believing this, the owner would accept offers $x \geq o - \phi$. For the taker to carry out the owner's belief, it has to be that $t - \frac{x+\phi+H}{2} - \phi >$

$-\psi$, which implies $x < 2t - H - 3\phi + 2\psi$. Since this threshold differs from the above condition for a never-take equilibrium, it follows that there is no equilibrium with a pure taker response in the interval $]2t - H - 3\phi + 2\psi, 2t - H - 2\phi + \psi[$, provided that at least some owners accept. The respective interval always exists since $\phi > \psi$.

To determine the taker's mixed strategy, let p be the probability that the taker infringes the right and let $\breve{o}$ be the cutoff value such that owners with $o \leq \breve{o}$ accept while higher-valuation owners reject.

For $\breve{o} \leq 0$, all owners reject. After being rejected, the taker's expected payoff is $p\left(t - \frac{H}{2} - \phi\right) - (1-p)\psi = p\left(t - \frac{H}{2} - \phi + \psi\right) - \psi$. Given this payoff, the taker would only be willing to randomize—set p between 0 and 1—if $t = \frac{H}{2} + \phi - \psi$. With this valuation, the offer has to satisfy $x < -\psi$ to remain below the upper limit of the mixed-strategy equilibrium. But a taker with this valuation is still an "ex ante respecter" so that all owners reject such an offer and the case falls under the all-reject equilibrium. Hence, we can rule out $\breve{o} \leq 0$.

With $\breve{o} > 0$, the taker's expected payoff after rejection is $p\left(t - \frac{\breve{o}+H}{2} - \phi\right) - (1-p)\psi = p\left(t + \psi - \frac{\breve{o}+H}{2} - \phi\right) - \psi$. The taker only randomizes if $t + \psi - \frac{\breve{o}+H}{2} - \phi = 0 \Leftrightarrow \breve{o} = 2t - H - 2\phi + 2\psi$. For $\breve{o}$ to constitute the cutoff, it must be that $x = p(\breve{o} - \phi) + (1-p)(\breve{o} - \psi) \Leftrightarrow p = \frac{\breve{o}-x-\psi}{\phi-\psi}$. Inserting the taker's randomization condition, we obtain $p = \frac{2t-H-2\phi+\psi-x}{\phi-\psi}$, which is between zero and one for offers in the interval of the mixed-strategy equilibrium. The taker's expected payoff then is

$$\Pi_T^M = \frac{\breve{o}}{H}(t-x) + \left(1 - \frac{\breve{o}}{H}\right)\left(p\left(t - \frac{\breve{o}+H}{2} - \phi + \psi\right) - \psi\right)$$

$$\Pi_T^M = x - t - 2\psi + \frac{2(t - x + \psi)(t - \phi + \psi)}{H}$$

Always-take equilibrium, $x \in [-\phi, 2t - H - 3\phi + 2\psi]$**.** We continue to consider only offers that some owners accept. If such an offer is below the lower bound of the mixed-strategy equilibrium, that is, $x \leq 2t - H - 3\phi + 2\psi$, the taker always infringes the entitlement if the owner rejects. This requires $t-\frac{x+\phi+H}{2}-\phi \geq -\psi$, which transforms into $x \leq 2t-H-3\phi+2\psi$. The resulting offer interval $[-\phi, 2t-H-3\phi+2\psi]$ is empty if $t < \frac{H}{2}+\phi-\psi$, that is, for "ex ante respecters". For other takers, the expected payoff is

$$\Pi_T^T = \frac{x+\phi}{H}(t-x) + \left(1-\frac{x+\phi}{H}\right)\left(t-\frac{x+\phi+H}{2}-\phi\right)$$

$$\Pi_T^T = t-\frac{H}{2}-\phi-\frac{(x-3\phi)(x+\phi)}{2H}$$

All-reject equilibrium, $x<-\phi$ or $x<-\psi$. For $x<-\phi$, all owners reject. As demonstrated in the discussion of the never-take equilibrium, owners also universally reject offers $x<-\psi$ if they face an "ex ante respecter" with $t \leq \frac{H}{2}+\phi-\psi$. In either case, after making an all-reject offer, takers abstain if they are ex ante respecters and appropriate the entitlement otherwise. Their expected payoff is

$$\Pi_T^R = \max\left(-\psi, t-\frac{H}{2}-\phi\right)$$

(2) The first stage: taker's offer

(a) Optimal offers within each range

We proceed by identifying the optimal offers within each of the offer ranges:

Optimal all-accept offer, $x \geq H-\psi$. Clearly, the taker never offers more than $x^A = H-\psi$. The resulting optimal payoff is $\Pi_T^A = t-H+\psi$.

Optimal never-take offers, $x \in [2t-H-2\phi+\psi, H-\psi]$. The payoff $\Pi_T^N = \frac{x+\psi}{H}(t-x+\psi)-\psi$ reaches an (interior) maximum at $\dot{x}^N = \frac{t}{2}$ with $\dot{\Pi}_T^N = \frac{\left(\frac{t}{2}+\psi\right)^2}{H} - \psi$. This maximum is above the upper bound of the relevant interval if $t > 2H-2\psi$ so that the border maximum with $\bar{x}^N = H-\psi$ and $\bar{\Pi}_T^N = t-H+\psi$ obtains. Conversely, if

$$\frac{t}{2} < 2t-H-2\phi+\psi$$

$$t > \frac{2}{3}(H + 2\phi - \psi)$$

an offer $\underline{x}^N = 2t - H - 2\phi + \psi$ at the lower border of the interval yields the maximum payoff with

$$\underline{\Pi}_T^N = \frac{2t - H - 2\phi + 2\psi}{H}(-t + H + 2\phi) - \psi$$

Optimal mixed-strategy offers, $x \in]2t - H - 3\phi + 2\psi, 2t - H - 2\phi + \psi[$**.** Because the payoff Π_T^M is a linear function of x the maximum is either at the upper or lower boundary. The taker never makes a mixed-strategy offer but either a never-take or an always-take offer.

Optimal always-take offers, $x \in [-\phi, 2t - H - 3\phi + 2\psi]$**.** The taker's payoff

$$\Pi_T^T = t - \frac{H}{2} - \phi - \frac{(x - 3\phi)(x + \phi)}{2H}$$

has an interior maximum at $\dot{x}^T = \phi$ with

$$\dot{\Pi}_T^T = t - \frac{H}{2} - \phi + \frac{2\phi^2}{H}$$

$\dot{x}^T$ is always above the lower bound of always-take offers. It is below the upper bound if $t \geq \frac{H}{2} + 2\phi - \psi$. Otherwise, there is a border maximum at $\bar{x}^T = 2t - H - 3\phi + 2\psi$ with

$$\overline{\Pi}_T^T = t - \frac{H}{2} - \phi - \frac{(2t - H - 6\phi + 2\psi)(2t - H - 2\phi + 2\psi)}{2H}$$

Optimal all-reject offers, $x < -\phi$ **or** $x < -\psi$**:** Since all offers in this range are rejected, the taker need not choose an optimal offer in this range. The payoff is always $\Pi_T^R = \max(-\psi, t - \frac{H}{2} - \phi)$.

(b) Eliminating mixed-strategy, all-accept, and all-reject offers

We continue by ruling out certain offer ranges: We have already disposed of mixed-strategy offers. Next, observe that the payoff from a never-take offer is weakly superior to that from an all-accept offer as the taker can always make the upper-limit never-take offer $\bar{x}^N$. As to all-reject offers, we first consider an "ex ante respecter" taker with $t \leq \frac{H}{2} + \phi - \psi$. Her payoff with an all-reject offer never exceeds $-\psi$. She can assure herself of the higher interior maximum payoff $\dot{\Pi}_T^N = \frac{\left(\frac{t}{2}+\psi\right)^2}{H} - \psi$ from a never-take offer if $t \leq 2H - 2\psi$. Otherwise, the upper-border maximum with $\overline{\Pi}_T^N = t - H + \psi$ applies and for $t > 2H - 2\psi$ also exceeds the payoff under all-reject offers. Turning to "ex ante infringer" takers with $t > \frac{H}{2} + \phi - \psi$, an all-reject offer gives her a payoff of $t - \frac{H}{2} - \phi$. By contrast, an always-take offer generates a higher interior maximum payoff $\dot{\Pi}_T^T = t - \frac{H}{2} - \phi + \frac{2\phi^2}{H}$ if $t \geq \frac{H}{2} + 2\phi - \psi$; in the opposite case, the taker prefers the border maximum of always-take to all-reject:

$$t - \frac{H}{2} - \phi < t - \frac{H}{2} - \phi - \frac{(2t - H - 6\phi + 2\psi)(2t - H - 2\phi + 2\psi)}{2H}$$

$$0 > (2t - H - 6\phi + 2\psi)(2t - H - 2\phi + 2\psi)$$

For $t \in]\frac{H}{2} + \phi - \psi, \frac{H}{2} + 2\phi - \psi[$, the first factor on the RHS is always positive while the second factor is negative. Therefore, the inequality holds and we can dismiss all-reject for the "ex ante infringer" type as well.

(c) Comparing payoffs from remaining strategies

The following table summarizes the payoffs for the remaining strategies and the relevant domains in terms of t. It also introduces the three relevant threshold values t^T, $\bar{t}^N$, and $\underline{t}^N$:

$\bar{x}^T =$ $2t - H - 3\phi + 2\psi$	$\bar{\Pi}_T^T = t - \frac{H}{2} - \phi - \frac{(2t - H - 6\phi + 2\psi)(2t - H - 2\phi + 2\psi)}{2H}$	$t < t^T$ $= \frac{H}{2} + 2\phi - \psi$
$\dot{x}^T = \phi$	$\dot{\Pi}_T^T = t - \frac{H}{2} - \phi + \frac{2\phi^2}{H}$	$t \geq t^T$
$\dot{x}^N = \frac{t}{2}$	$\dot{\Pi}_T^N = \frac{\left(\frac{t}{2} + \psi\right)^2}{H} - \psi$	$t \leq \bar{t}^N$ $= 2H - 2\psi \;\; \wedge$ $t \leq \underline{t}^N$ $= \frac{2}{3}(H + 2\phi - \psi)$
$\bar{x}^N = H - \psi$	$\bar{\Pi}_T^N = t - H + \psi$	$t > \bar{t}^N$
$\underline{x}^N = 2t - H - 2\phi + \psi$	$\underline{\Pi}_T^N = \frac{2t - H - 2\phi + 2\psi}{H}(-t + H + 2\phi) - \psi$	$t > \underline{t}^N$

Note that the three thresholds t^T, $\bar{t}^N$, and $\underline{t}^N$ are greater than zero. To narrow down which strategies we need to compare, we determine the relations between the three thresholds: Firstly, $\bar{t}^N > \underline{t}^N$ if

$$2H - 2\psi > \frac{2}{3}(H + 2\phi - \psi)$$

$$H > \phi + \psi$$

Secondly, $\underline{t}^N > t^T$ if

$$\frac{2}{3}(H + 2\phi - \psi) > \frac{H}{2} + 2\phi - \psi$$

$$H > 4\phi - 2\psi$$

Lastly, $\bar{t}^N > t^T$ if

$$2H - 2\psi > \frac{H}{2} + 2\phi - \psi$$

$$H > \frac{4}{3}\phi + \frac{2}{3}\psi$$

Because $\phi + \psi < \frac{4}{3}\phi + \frac{2}{3}\psi < 4\phi - 2\psi$, we are left with the following cases:

If $H \leq \phi + \psi$	$\bar{t}^N \leq \underline{t}^N < t^T$
If $\phi + \psi < H \leq \frac{4}{3}\phi + \frac{2}{3}\psi$	$\underline{t}^N \leq \bar{t}^N < t^T$
If $\frac{4}{3}\phi + \frac{2}{3}\psi < H \leq 4\phi - 2\psi$	$\underline{t}^N < t^T \leq \bar{t}^N$
If $H > 4\phi - 2\psi$	$t^T < \underline{t}^N < \bar{t}^N$

The following Figure 10 illustrates which offers need to be compared.

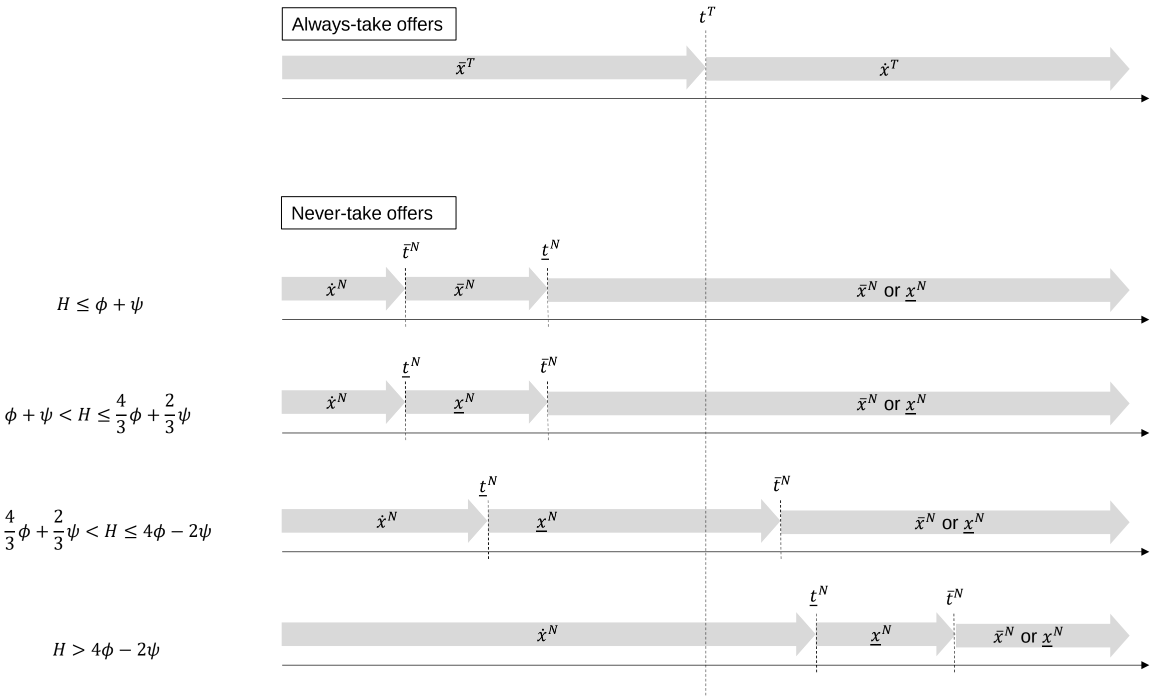

Figure 10: Relevant combinations of optimal offers

We proceed by comparing the relevant combinations between the remaining five offers $\bar{x}^T$, $\dot{x}^T$, $\dot{x}^N$, $\bar{x}^N$, and $\underline{x}^N$ for their respective domains.

Upper maximum always-take vs. interior maximum never-take. To determine which of the offers $\bar{x}^T$ and $\dot{x}^N$ provides the taker with the highest payoff, we define the function $\Delta\bar{\Pi}_T^T\dot{\Pi}_T^N(t) = \bar{\Pi}_T^T - \dot{\Pi}_T^N$:

$$\Delta\bar{\Pi}_T^T\dot{\Pi}_T^N(t) = t - \frac{H}{2} - \phi - \frac{(2t - H - 6\phi + 2\psi)(2t - H - 2\phi + 2\psi)}{2H} - \frac{\left(\frac{t}{2} + \psi\right)^2}{H} + \psi$$

The function is continuous, twice differentiable, and has a single maximum. Setting $\Delta\bar{\Pi}_T^T\dot{\Pi}_T^N(t) = 0$ yields two cutoffs

$$t_{1,2}^{\bar{T}\dot{N}} = \frac{2}{9}\left(3H + 8\phi - 5\psi \pm \sqrt{(\phi - \psi)(3H + 10\phi + 2\psi)}\right)$$

The upper cutoff $t_1^{\bar{T}\dot{N}}$ can be ignored because it is above $\underline{t}^N$ (where $\dot{x}^N$ is already inferior to $\underline{x}^N$). We therefore drop the subscript and write only $t^{\bar{T}\dot{N}}$:

$$t^{\bar{T}\dot{N}} = \frac{2}{9}\left(3H + 8\phi - 5\psi - \sqrt{(\phi - \psi)(3H + 10\phi + 2\psi)}\right)$$

$t^{\bar{T}\dot{N}}$ is always above zero and below $\underline{t}^N$. It is below $\bar{t}^N$ only if

$$0 > 4\phi + 2\psi - 3H - \frac{1}{2}\sqrt{(\phi - \psi)(3H + 10\phi + 2\psi)}$$

The RHS as a function of H is continuous, the first derivative is strictly negative throughout. Given that $0 < \psi < \phi$, the only root is at

$$H^I = \frac{11}{8}\phi + \frac{5}{8}\psi - \frac{1}{8}\sqrt{25\phi^2 - 18\phi\psi - 7\psi^2}$$

It follows that $t^{\bar{T}\dot{N}}$ remains below $\bar{t}^N$ if $H > H^I$.

Finally, $t^{\bar{T}\dot{N}}$ needs to be below t^T in order to be in the relevant domain. This is the case if

$$0 < 4\phi + 2\psi - 3H + 4\sqrt{(\phi - \psi)(3H + 10\phi + 2\psi)}$$

Again, the RHS as a function of H is continuous and has a strictly negative first derivative. The only intercept is at

$$H^{II} = 4\phi - 2\psi + 4\sqrt{2}\sqrt{\phi^2 - \phi\psi}$$

Hence, $t^{\bar{T}\dot{N}}$ is below t^T if $H < H^{II}$.

Note that $H^I < H^{II}$, so that $t^{\bar{T}\dot{N}}$ is in the relevant range for $H^I < H < H^{II}$. In this case, $\bar{x}^T$ is preferred to $\dot{x}^N$ if $t > t^{\bar{T}\dot{N}}$.

Upper maximum always-take vs. upper maximum never-take. For the choice between offers $\bar{x}^T$ and $\bar{x}^N$ we again define a function $\Delta\overline{\Pi}_T^T\overline{\Pi}_T^N(t)$ with

$$\Delta\overline{\Pi}_T^T\overline{\Pi}_T^N(t) = t - \frac{H}{2} - \phi - \frac{(2t - H - 6\phi + 2\psi)(2t - H - 2\phi + 2\psi)}{2H} - t + H - \psi$$

$$\Delta\overline{\Pi}_T^T\overline{\Pi}_T^N(t) = \frac{H}{2} - \phi - \psi - \frac{(2t - H - 6\phi + 2\psi)(2t - H - 2\phi + 2\psi)}{2H}$$

Again, the function is continuous, concave, and has a maximum. The function is valued zero at

$$t_{1,2}^{\bar{T}\bar{N}} = \frac{H}{2} + 2\phi - \psi \pm \frac{1}{2}\sqrt{H^2 - 2H\phi + 4\phi^2 - 2H\psi}$$

We can ignore the upper cutoff $t_1^{\bar{T}\bar{N}}$ because it exceeds H. The only relevant cutoff is

$$t^{\bar{T}\bar{N}} = \frac{H}{2} + 2\phi - \psi - \frac{1}{2}\sqrt{H^2 - 2H\phi + 4\phi^2 - 2H\psi}$$

$t^{\bar{T}\bar{N}}$ is always greater than zero and below t^T. It is above $\bar{t}^N$ if

$$0 < 4\phi + 2\psi - 3H - \sqrt{H^2 - 2H\phi + 4\phi^2 - 2H\psi}$$

We apply the same procedure as before to find that $t^{\bar{T}\bar{N}} > \bar{t}^N$ if

$$H < H^I$$

Hence, $\bar{x}^T$ is preferred to $\bar{x}^N$ if $t > t^{\bar{T}\bar{N}}$. The threshold $t^{\bar{T}\bar{N}}$ is in the relevant domain (i.e., above $\bar{t}^N$) if $H < H^I$.

Upper maximum always-take vs. lower maximum never-take. We again define a function $\Delta\overline{\Pi}_T^T\underline{\Pi}_T^N(t)$ to choose between offers $\bar{x}^T$ and $\underline{x}^N$:

$$\Delta\overline{\Pi}_T^T\underline{\Pi}_T^N(t) = t - \frac{H}{2} - \phi - \frac{(2t - H - 6\phi + 2\psi)(2t - H - 2\phi + 2\psi)}{2H} - \frac{2t - H - 2\phi + 2\psi}{H}(-t + H + 2\phi) + \psi$$

$$\Delta\overline{\Pi}_T^T\underline{\Pi}_T^N(t) = \frac{(\phi - \psi)(2t - H + 2\phi - 2\psi}{H}$$

This increasing linear function equals zero at

$$t^{\bar{T}\underline{N}} = \frac{H}{2} + \phi - \psi$$

Since $t^{\bar{T}\underline{N}}$ is below $\underline{t}^N$, $\bar{x}^T$ is superior to $\underline{x}^N$ in the relevant range, for $\underline{t}^N < t < t^T$.

Interior maximum always-take vs. interior maximum never-take. The choice between $\dot{x}^T$ and $\dot{x}^N$ is governed by $\Delta\dot{\Pi}_T^T\dot{\Pi}_T^N(t)$:

$$\Delta\dot{\Pi}_T^T\dot{\Pi}_T^N(t) = t - \frac{H}{2} - \phi + \frac{2\phi^2}{H} - \frac{\left(\frac{t}{2} + \psi\right)^2}{H} + \psi$$

$$\Delta\dot{\Pi}_T^T\dot{\Pi}_T^N(t) = t - \frac{H}{2} - \phi + \psi + \frac{2\phi^2 - \left(\frac{t}{2} + \psi\right)^2}{H}$$

The function is once more continuous, concave, and has a maximum. The roots are at

$$t_{1,2}^{\dot{T}\dot{N}} = 2H - 2\psi \pm \sqrt{2}\sqrt{H^2 - 2H\phi + 4\phi^2 - 2H\psi}$$

where $t_1^{\dot{T}\dot{N}} > H$ so that we are left with the only relevant cutoff

$$t^{\dot{T}\dot{N}} = 2H - 2\psi - \sqrt{2}\sqrt{H^2 - 2H\phi + 4\phi^2 - 2H\psi}$$

$t^{\dot{T}\dot{N}}$ is below $\underline{t}^N$ and $\bar{t}^N$. However, it is above t^T and hence in the domain of $\dot{x}^T$ only if

$$0 > 4\phi + 2\psi - 3H + 2\sqrt{2}\sqrt{H^2 - 2H\phi + 4\phi^2 - 2H\psi}$$

Again using the above approach, this translates into the condition $H > H^{II}$.

Hence, $\dot{x}^T$ is superior to $\dot{x}^N$ for $t < t^{\dot{T}\dot{N}}$ and $H > H^{II}$.

Interior maximum always-take vs. upper maximum never-take. $\dot{x}^T$ is superior to $\bar{x}^N$ if $\dot{\Pi}_T^T > \overline{\Pi}_T^N$:

$$t - \frac{H}{2} - \phi + \frac{2\phi^2}{H} > t - H + \psi$$

$$\frac{H}{2} + \frac{2\phi^2}{H} > \phi + \psi$$

$$H^2 > 2H(\phi + \psi) - 4\phi^2$$

Substituting ϕ for ψ and finding the maximum of the RHS in terms of ϕ shows that this equality never holds. Hence, $\dot{x}^T$ is superior to $\bar{x}^N$.

Interior maximum always-take vs. upper maximum never-take. $\dot{x}^T$ also yields a higher payoff than $\underline{x}^N$:

$$t - \frac{H}{2} - \phi + \frac{2\phi^2}{H} > \frac{2t - H - 2\phi + 2\psi}{H}(-t + H + 2\phi) - \psi$$

$$2Ht - H^2 - 2H\phi + 4\phi^2 > -(2t - H - 2\phi + 2\psi)(2t - 2H - 4\phi) - 2H\psi$$

$$H^2 - 4Ht + 4t^2 + 6H\phi - 12t\phi + 12\phi^2 - 2H\psi + 4t\psi - 8\phi\psi > 0$$

The inequality is hardest to satisfy for $t = H$:

$$H^2 - 6H\phi + 12\phi^2 + 2H\psi - 8\phi\psi > 0$$

This latter inequality, in turn, is hardest to satisfy for $\psi = \phi$ if $8\phi > 2H$:

$$H^2 - 4H\phi + 4\phi^2 > 0$$

$$(H - 2\phi)^2 > 0$$

which is true.

If $8\phi > 2H$ the inequality is hardest to satisfy for $\psi = 0$:

$$H^2 - 6H\phi + 12\phi^2 > 0$$

$$H^2 - 6H\phi + 9\phi^2 + 3\phi^2 > 0$$

$$(H - 3\phi)^2 + 3\phi^2 > 0$$

which is true. Hence, $\dot{x}^T$ is always superior to $\underline{x}^N$.

Upper maximum never-take vs. lower maximum never-take. Finally, if t exceeds both $\underline{t}^N$ and $\bar{t}^N$, we need to determine whether the taker prefers $\bar{x}^N$ to $\underline{x}^N$. We again define a function

$$\Delta\overline{\Pi}_T^N\underline{\Pi}_T^N(t) = t - H + \psi - \frac{2t - H - 2\phi + 2\psi}{H}(-t + H + 2\phi) + \psi$$

The function is continuous and twice differentiable. The second derivative is $\frac{4}{H}$, the minimum is at $t = \frac{H}{2} + \frac{3}{2}\phi - \frac{1}{2}\psi$. The function takes on the value zero for $t_1^{\bar{N}\underline{N}} = H + \phi - \psi$, which we can disregard because it exceeds H. The other root is $t_2^{\bar{N}\underline{N}} = 2\phi$. If $t_2^{\bar{N}\underline{N}} \geq t_1^{\bar{N}\underline{N}} \Leftrightarrow H \leq \phi + \psi$, then $\Delta\overline{\Pi}_T^N\underline{\Pi}_T^N(t) > 0$ for all $t \leq H$. Conversely, for $t_2^{\bar{N}\underline{N}} < t_1^{\bar{N}\underline{N}} \Leftrightarrow H > \phi + \psi$, it turns out that $t_2^{\bar{N}\underline{N}}$ is below both $\underline{t}^N$ and $\bar{t}^N$. Hence, $\Delta\overline{\Pi}_T^N\underline{\Pi}_T^N(t) < 0$ in the relevant range.

As a result, $\bar{x}^N$ is superior to $\underline{x}^N$ for $H \leq \phi + \psi$, and inferior otherwise.

(d) Equilibrium offers

The following table summarizes the results:

$\bar{x}^T \succ \dot{x}^N$	if $t > t^{\bar{T}\dot{N}} = \frac{2}{9}\left(3H + 8\phi - 5\psi - \sqrt{(\phi-\psi)(3H + 10\phi + 2\psi)}\right)$ $t^{\bar{T}\dot{N}} < t^T$ if $H < H^{II} = 4\phi - 2\psi + 4\sqrt{2}\sqrt{\phi^2 - \phi\psi}$ $t^{\bar{T}\dot{N}} < \underline{t}^N$ always holds $t^{\bar{T}\dot{N}} < \bar{t}^N$ if $H > H^I = \frac{11}{8}\phi + \frac{5}{8}\psi - \frac{1}{8}\sqrt{25\phi^2 - 18\phi\psi - 7\psi^2}$
$\bar{x}^T \succ \bar{x}^N$	if $t > t^{\bar{T}\bar{N}} = \frac{H}{2} + 2\phi - \psi - \frac{1}{2}\sqrt{H^2 - 2H\phi + 4\phi^2 - 2H\psi}$ $t^{\bar{T}\bar{N}} < t^T$ always holds $t^{\bar{T}\bar{N}} > \bar{t}^N$ if $H < H^I$
$\bar{x}^T \succ \underline{x}^N$	always holds
$\dot{x}^T \succ \dot{x}^N$	if $t > t^{\dot{T}\dot{N}} = 2H - 2\psi - \sqrt{2}\sqrt{H^2 - 2H\phi + 4\phi^2 - 2H\psi}$ $t^{\dot{T}\dot{N}} > t^T$ if $H > H^{II}$ $t^{\dot{T}\dot{N}} < \underline{t}^N$ and $t^{\dot{T}\dot{N}} < \bar{t}^N$ always hold.
$\dot{x}^T \succ \bar{x}^N$	always holds
$\dot{x}^T \succ \underline{x}^N$	always holds
$\bar{x}^N \succ \underline{x}^N$	if $H < \phi + \psi$

Note that the two cutoffs H^I and H^{II} align with the thresholds of Figure 10 in that $H^I < \phi + \psi$ and $H^{II} > 4\phi - 2\psi$. The following collates the equilibrium offers for the various ranges of H. Proposition 2 reflects these results, collapsing the cases (II)–(V) below into one.

(I) For $H \leq H^I = \frac{11}{8}\phi + \frac{5}{8}\psi - \frac{1}{8}\sqrt{25\phi^2 - 18\phi\psi - 7\psi^2}$:

Takers with $t \leq \bar{t}^N$ offer $\dot{x}^N$.

Takers with $\bar{t}^N < t \leq t^{\bar{T}\bar{N}} = \frac{H}{2} + 2\phi - \psi - \frac{1}{2}\sqrt{H^2 - 2H\phi + 4\phi^2 - 2H\psi}$ offer $\bar{x}^N$.

Takers with $t^{\bar{T}\bar{N}} < t \leq t^T$ offer $\bar{x}^T$.

We need not consider greater t because they cannot occur if $H \leq H^I$. To see this, remember that $t \leq H$. For a t to exceed t^T, we need $H > t^T = \frac{H}{2} + 2\phi - \psi \Leftrightarrow H > 4\phi - 2\psi$. But because $H \leq H^I = \frac{11}{8}\phi + \frac{5}{8}\psi - \frac{1}{8}\sqrt{25\phi^2 - 18\phi\psi - 7\psi^2}$ it would have to be that $4\phi - 2\psi < H^I$, which does not hold.

(II) For $H^I < H \leq \phi + \psi$:

Takers with $t \leq t^{\bar{T}\dot{N}} = \frac{2}{9}\left(3H + 8\phi - 5\psi - \sqrt{(\phi - \psi)(3H + 10\phi + 2\psi)}\right)$ offer $\dot{x}^N$.

Takers with $t^{\bar{T}\dot{N}} < t \leq t^T$ offer $\bar{x}^T$.

Takers with $t > t^T$ offer $\dot{x}^T$.

(III) For $\phi + \psi < H \leq \frac{4}{3}\phi + \frac{2}{3}\psi$: Same as case (II).

(IV) For $\frac{4}{3}\phi + \frac{2}{3}\psi < H \leq 4\phi - 2\psi$: Same as case (II).

(V) For $4\phi - 2\psi < H \leq H^{II} = 4\phi - 2\psi + 4\sqrt{2}\sqrt{\phi^2 - \phi\psi}$: Same as case (II).

(VI) For $H > H^{II}$

Takers with $t \leq t^{\dot{T}\dot{N}} = 2H - 2\psi - \sqrt{2}\sqrt{H^2 - 2H\phi + 4\phi^2 - 2H\psi}$ offer $\dot{x}^N$.

Takers with $t > t^{\dot{T}\dot{N}}$ offer $\dot{x}^T$.

c) Proposition 3. Welfare comparison of the property and liability rules

To study the welfare implications, we consider the owner's payoff as an expected value over owner types. For the property rule and the taker's equilibrium offers, we have

$x = \frac{t}{2}$	$\mathrm{E}_o\left(\dot{\Pi}_{O\,PR}\right) = \frac{\frac{t}{2}+\psi}{H}\frac{t}{2} + \left(1 - \frac{\frac{t}{2}+\psi}{H}\right)\left(\frac{\frac{t}{2}+\psi+H}{2} - \psi\right) = \frac{H}{2} - \psi + \frac{(t+2\psi)^2}{8H}$
$x = H - \psi$	$\mathrm{E}_o(\overline{\Pi}_{O\,PR}) = H - \psi$

For the liability rule we obtain

$\dot{x}^N = \frac{t}{2}$	$\mathrm{E}_o\left(\dot{\Pi}^N_{O\,LR}\right) = \mathrm{E}_o\left(\dot{\Pi}_{O\,PR}\right) = \frac{H}{2} - \psi + \frac{(t+2\psi)^2}{8H}$
$\bar{x}^N = H - \psi$	$\mathrm{E}_o(\overline{\Pi}^N_{O\,LR}) = \mathrm{E}_o(\overline{\Pi}_{O\,PR}) = H - \psi$
$\bar{x}^T = 2t - H - 3\phi + 2\psi$	$\mathrm{E}_o(\overline{\Pi}^T_{O\,LR}) = \frac{\bar{x}^T + \phi}{H}\bar{x}^T + \left(1 - \frac{\bar{x}^T + \phi}{H}\right)\left(\frac{\bar{x}^T + \phi + H}{2} - \phi\right)$ $= H - 2t + \phi - 2\psi + \frac{2(t - \phi + \psi)^2}{H}$
$\dot{x}^T = \phi$	$\mathrm{E}_o\left(\dot{\Pi}^T_{O\,LR}\right) = \frac{2\phi}{H}\phi + \left(1 - \frac{2\phi}{H}\right)\left(\frac{2\phi+H}{2} - \phi\right) = \frac{H}{2} - \phi + \frac{2\phi^2}{H}$

As to the taker's payoffs, we refer to VII.1.b)(2)(c) above.

Calculating the total (expected) payoffs for the various equilibrium offers under the property rule and the liability rule:

$$\dot{\Pi}_{PR} = \mathrm{E}_o\left(\dot{\Pi}_{O\,PR}\right) + \dot{\Pi}_{T\,PR} = \frac{H}{2} - \psi + \frac{(t+2\psi)^2}{8H} + \frac{\left(\frac{t}{2}+\psi\right)^2}{H} - \psi = \frac{H}{2} - 2\psi + \frac{3}{8}\frac{(t+2\psi)^2}{H}$$

$$\overline{\Pi}_{PR} = \mathrm{E}_o(\overline{\Pi}_{O\,PR}) + \overline{\Pi}_{T\,PR} = H - \psi + t - H + \psi = t$$

$$\dot{\Pi}^N_{LR} = \dot{\Pi}_{PR} = \frac{H}{2} - 2\psi + \frac{3}{8}\frac{(t+2\psi)^2}{H}$$

$$\overline{\Pi}^N_{LR} = \overline{\Pi}_{PR} = t$$

$$\overline{\Pi}^T_{LR} = \mathrm{E}_o(\overline{\Pi}^T_{O\;LR}) + \overline{\Pi}^T_{T\;LR} =$$

$$H - 2t + \phi - 2\psi + \frac{2(t - \phi + \psi)^2}{H} + t - \frac{H}{2} - \phi$$

$$- \frac{(2t - H - 6\phi + 2\psi)(2t - H - 2\phi + 2\psi)}{2H} =$$

$$t + 4\phi\left(\frac{t - \phi + \psi}{H} - 1\right)$$

$$\dot{\Pi}^T_{LR} = \mathrm{E}_o\left(\dot{\Pi}^T_{O\;LR}\right) + \dot{\Pi}^T_{T\;LR} = \frac{H}{2} - \phi + \frac{2\phi^2}{H} + t - \frac{H}{2} - \phi + \frac{2\phi^2}{H} = t - 2\phi + \frac{4\phi^2}{H}$$

Proposition 3 reflects comparisons of total payoffs under the two rules as a function of t. The relevant combinations reflect the equilibrium ranges from Propositions 1 and 2. The only t threshold in Proposition 1 (property rule) is $2H - 2\psi$, which equals $\bar{t}^N$ from Proposition 2 (liability rule). Inspecting the relations between the various cutoffs in Proposition 2 from above, we arrive at Figure 11 for the relevant cases.

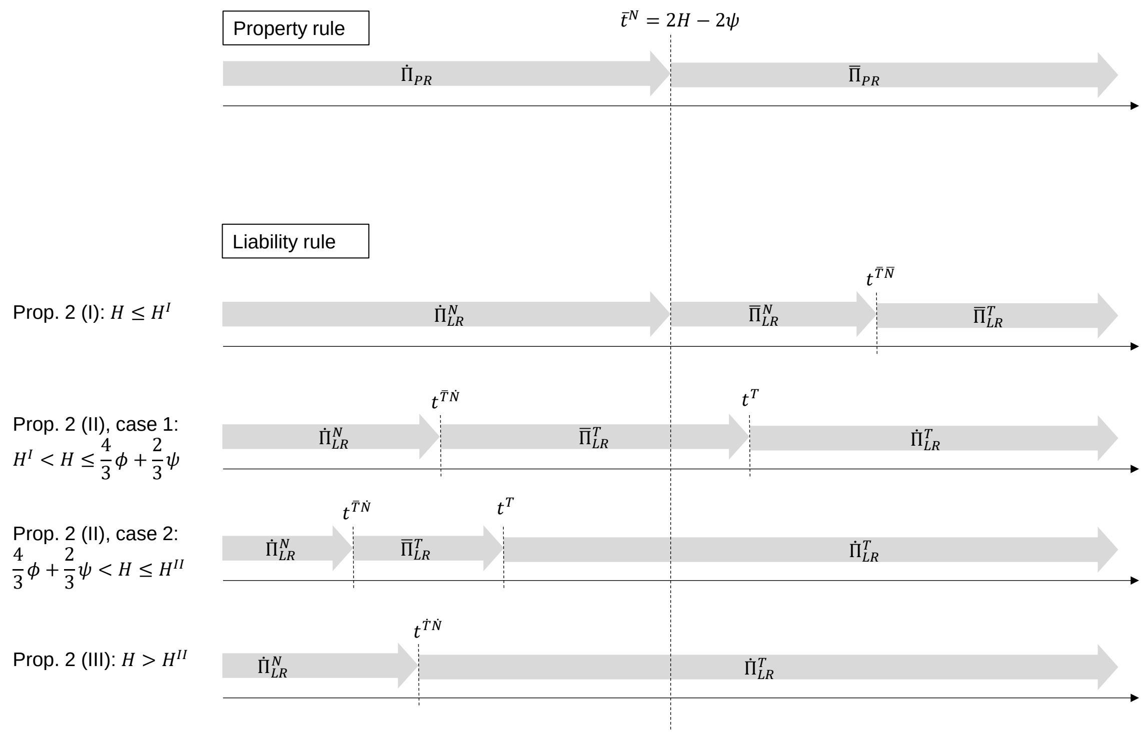

Figure 11: Relevant combinations of total equilibrium payoffs

I am not sure whether $t = 2H - 2\psi$ can be a relevant cutoff for the Liability Rule, Propoition 2 (II) case 2 and Proposition 2 (III), because it would be necessary that $2\psi > H$. That means:

Proposition 2 (III)

$$H^{II} = 4\phi - 2\psi + 4\sqrt{2}\sqrt{\phi^2 - \phi\psi} < H < 2\psi$$

$$4\phi - 2\psi + 4\sqrt{2}\sqrt{\phi^2 - \phi\psi} < 2\psi$$

$$\phi + \sqrt{2}\sqrt{\phi^2 - \phi\psi} < \psi$$

This never holds.

Proposition 2 (II) case 2

$$\frac{4}{3}\phi + \frac{2}{3}\psi < H < 2\psi$$

$$\phi < \psi$$

which is never true by assumption.

Proposition 2 (II) case 1

$$H^{I} = \frac{11}{8}\phi + \frac{5}{8}\psi - \frac{1}{8}\sqrt{25\phi^2 - 18\phi\psi - 7\psi^2} < H < 2\psi$$

$$\frac{11}{8}\phi + \frac{5}{8}\psi - \frac{1}{8}\sqrt{25\phi^2 - 18\phi\psi - 7\psi^2} < 2\psi$$

$$11\phi - 11\psi - \sqrt{25\phi^2 - 18\phi\psi - 7\psi^2} < 0$$

$$\frac{3}{4}\phi < \psi < \phi$$

We know that $\dot{\Pi}_{PR} = \dot{\Pi}_{LR}^{N}$ and $\overline{\Pi}_{PR} = \overline{\Pi}_{LR}^{N}$.

Interior equilibrium property rule vs. upper always-take equilibrium liability rule.

$$\dot{\Pi}_{PR} \geq \overline{\Pi}_{LR}^{T}$$

$$\frac{H}{2} - 2\psi + \frac{3}{8}\frac{(t+2\psi)^2}{H} \geq t + 4\phi\left(\frac{t-\phi+\psi}{H} - 1\right)$$

$$O \geq t - \frac{H}{2} + 2\psi + 4\phi\left(\frac{t-\phi+\psi}{H} - 1\right) - \frac{3}{8}\frac{(t+2\psi)^2}{H}$$

The RHS as a function of t is concave, always has a maximum and two roots at $t = \frac{4}{3}H + \frac{16}{3}\phi - 2\psi \pm \frac{2}{3}\sqrt{H^2 + 8H\phi + 40\phi^2 - 24\phi\psi}$. The higher root always exceeds $\bar{t}^N = 2H - 2\psi$ and thus is outside the relevant range. The lower root is below $\bar{t}^N$ if $H > \phi + \psi$ (Proposition 2 (II), case 1 in Figure 11) and below t^T if $H < \frac{10}{3}\psi - 4\phi + \frac{8}{3}\sqrt{6\phi^2 - 6\phi\psi + \psi^2}$ (Proposition 2 (II), case 1). Hence, for the lower root to be in the relevant range, we need $\phi + \psi < H < \frac{10}{3}\psi - 4\phi + \frac{8}{3}\sqrt{6\phi^2 - 6\phi\psi + \psi^2}$. But when this condition is satisfied, the lower root exceeds H and never becomes binding. It follows that $\dot{\Pi}_{PR} \geq \overline{\Pi}_{LR}^{T}$ whenever the two equilibria combine.

Interior equilibrium property rule vs. interior always-take equilibrium liability rule.

$$\dot{\Pi}_{PR} \geq \dot{\Pi}_{LR}^{T}$$

$$\frac{H}{2} - 2\psi + \frac{3}{8}\frac{(t+2\psi)^2}{H} \geq t - 2\phi + \frac{4\phi^2}{H}$$

$$0 \geq t - \frac{H}{2} - 2\phi + 2\psi + \frac{4\phi^2}{H} - \frac{3}{8}\frac{(t+2\psi)^2}{H}$$

The RHS as a function of t is concave throughout and always has a maximum. It has roots at $t_{1,2}^{\dot{P}\dot{T}} = \frac{4}{3}H - 2\psi \pm \frac{2}{3}\sqrt{H^2 - 12H\phi + 24\phi^2}$, which exist only for $H \notin \left]\left(6 - 2\sqrt{3}\right)\phi, \left(6 + 2\sqrt{3}\right)\phi\right[$. If the RHS has no roots, the inequality is satisfied and the property rule dominates.

For $H \leq H^{II}$ (that is, Proposition 2 (II) case 2 in Figure 11), the two equilibria only coincide if $H \geq t^T$, which leads to $H \geq 4\phi - 2\psi$. The interval $[4\phi - 2\psi, (6 - 2\sqrt{3})\phi]$ is non-empty only if $\phi \leq \frac{\psi}{\sqrt{3}-1}$; but then $t_{1,2}^{\dot{P}\dot{T}} < t^T$, implying that the inequality holds and the property rule prevails. For $H > (6 + 2\sqrt{3})\phi$, the higher root $t_1^{\dot{P}\dot{T}}$ exceeds H and can be ignored; for $H \leq H^{II}$, the same is true for the lower root $t_2^{\dot{P}\dot{T}}$. It follows that the property rule is superior in these instances as well.

Turning to $H > H^{II}$ (Proposition 2 (III) in Figure 11), the interval $]H^{II}, (6 - 2\sqrt{3})\phi]$ is non-empty only if $\phi < \left(\frac{13}{2} - \frac{7\sqrt{3}}{2} + \sqrt{80 - 46\sqrt{3}}\right)\psi \approx 1.008\psi$. But in this case the roots are below the lower t limit for the interior always-take equilibrium under the liability rule, that is, $t_{1,2}^{\dot{P}\dot{T}} < t^{\dot{T}\dot{N}}$. Thus, the inequality holds and the property rule is preferred. The same holds true for $](6 - 2\sqrt{3})\phi, (6 + 2\sqrt{3})\phi[$, where RHS has no roots and its maximum is negative. For $H \geq (6 + 2\sqrt{3})\phi$, the higher root $t_1^{\dot{P}\dot{T}}$ exceeds H. As to the lower root, $t_2^{\dot{P}\dot{T}} \leq H$ if and only if $H \geq 8\phi - 2\psi + 4\sqrt{2\phi^2 - 2\phi\psi + \psi^2} = H^{III}$. It follows that the property rule prevails except for $H \geq H^{III}$ and $t > t_2^{\dot{P}\dot{T}} = \frac{4}{3}H - 2\psi - \frac{2}{3}\sqrt{H^2 - 12H\phi + 24\phi^2}$. In the Proposition, we refer to $t_2^{\dot{P}\dot{T}}$ as t_{PRLR}.

Upper equilibrium property rule vs. upper always-take equilibrium liability rule.

$$\overline{\Pi}_{PR} \geq \overline{\Pi}_{LR}^T$$

$$t \geq t + 4\phi\left(\frac{t - \phi + \psi}{H} - 1\right)$$

$$H + \phi - \psi \geq t$$

Because $t \leq H$, the inequality holds and the property rule always prevails in this pairing of equilibrium offers.

Upper equilibrium property rule vs. interior always-take equilibrium liability rule.

$$\overline{\Pi}_{PR} \geq \dot{\Pi}_{LR}^T$$

$$t \geq t - 2\phi + \frac{4\phi^2}{H}$$

$$H \geq 2\phi$$

The two equilibria occur under Proposition 2 (II) for $t > t^T$. For this case to arise, we need $t^T \leq H$:

$$\frac{H}{2} + 2\phi - \psi \leq H$$

$$4\phi - 2\psi \leq H$$

This inequality only holds for $H > \phi$, that is, when the property rule is superior.

The two equilibria also coincide for $H > H^{II}$ (and $t > \bar{t}^N$, Proposition 2 (III)):

$$H > H^{II}$$

$$H > 4\phi - 2\psi + 4\sqrt{2}\sqrt{\phi^2 - \phi\psi}$$

If the liability rule were to be superior, it would have to be $H < 2\phi$. Inserting $H = 2\phi$ in the above inequality gives us

$$2\phi > 4\phi - 2\psi + 4\sqrt{2}\sqrt{\phi^2 - \phi\psi}$$

$$2\psi > 2\phi + 4\sqrt{2}\sqrt{\phi^2 - \phi\psi}$$

which cannot be satisfied. It follows that whenever the two equilibria compare the property rule is preferred.

2. Signaling model

a) Proposition 4. Equilibrium of the signaling game under the property rule

The taker accepts any offer $x \leq t + \psi$ and rejects higher offers. The owner prefers acceptance over performance if $x - o \geq -\psi \Leftrightarrow x \geq o - \psi$. Knowing the taker's valuation, the highest acceptable demand is $x = t + \psi$. It follows that the owner demands $x = t + \psi$ if $o - \psi \leq t + \psi$, which the taker accepts. If the owner's valuation is higher, $o - \psi > t + \psi$, the owner prefers the taker to respect his entitlement. Hence, he makes an inacceptable offer $x > t + \psi$.

b) Proposition 5. Equilibria of the signaling game under the liability rule

(1) Case (I), $t \leq 2\phi$: Separating equilibrium, pure taker strategies

Consider for this equilibrium the owner's belief that the taker respects the entitlement if she rejects an offer. The taker prefers accept over reject-respecting for $t - x \geq -\psi \Leftrightarrow x \leq t + \psi$ and reject-respecting over accept for $t - x < -\psi \Leftrightarrow x > t + \psi$. It follows that owners with $o - \psi \leq t + \psi$ demand $x = t + \psi$.

Note for the upper limit of the equilibrium that for the taker to accept $x \leq t + \psi$, she also must be better off than with reject-take.

$$t - E(o|x = t + \psi) + \phi = t - \frac{t + \psi + \psi}{2} - \phi \leq t - x = -\psi \Leftrightarrow t \leq 2\phi$$

The owner's belief that the taker respects the entitlement after receiving demands $x > t + \psi$ is consistent because the taker always prefers to respect the entitlement to take:

$$t - E(o|x > t + \psi) - \phi = t - \frac{t + 2\psi + H}{2} - \phi < -\psi$$

$$H > t - 2\phi$$

(2) Case (II), $2\phi < t \leq 2\phi \ln\frac{2\phi}{\phi-\psi} + \phi - \psi = \bar{o} + \phi - \psi$: Separating equilibrium, mixed taker strategy

First note that a mixed strategy of accept and reject-take requires the taker to be indifferent between the two alternatives, i.e., $x = E(o|x) + \phi$. This is true if the owner demands $x = o + \phi$.

The taker's acceptance probability $p(x)$ must be such that the owner's demand $x = o + \phi$ maximizes his expected payoff. The owner's problem is:

$$\max_x p(x)(x) + \big(1 - p(x)\big)(o - \phi)$$

The first-order condition is:

$$p'(x)(x) + p(x) - p'(x)(o - \phi) = 0$$

$$\frac{p(x)}{p'(x)} = o - \phi - x \tag{1}$$

Inserting the separating demand $x = o + \phi$ into (1) we obtain:

$$-\frac{1}{2\phi}p(x) = p'(x)$$

$$-\frac{1}{2\phi} = \frac{p'(x)}{p(x)} = \frac{d}{dx}\ln(p(x))$$

Taking the infinite integral for both side we get:

$$-\frac{1}{2\phi}x + k = \ln(p(x))$$

$$e^{-\frac{x}{2\phi}+k} = p(x)$$

$$p(x) = ke^{-\frac{x}{2\phi}}$$

Since $p \in [0,1]$, p is a stepwise function:

$$p(x) = \begin{cases} 1 & x \leq \underline{x} \\ ke^{-\frac{x}{2\phi}} & x > \underline{x} \end{cases}$$

or, alternatively,

$$p(x) = \begin{cases} 1 & x \leq \underline{x} \\ ke^{-\frac{x}{2\phi}} & \underline{x} < x < \bar{x} \\ 0 & x \geq \bar{x} \end{cases}$$

k must be chosen such that $p \in [0,1]$. This implies:

$$ke^{-\frac{\underline{x}}{2\phi}} \leq 1 \Leftrightarrow k \leq e^{\frac{\underline{x}}{2\phi}}$$

and $ke^{-\frac{\bar{x}}{2\phi}} \geq 0 \Leftrightarrow k \geq 0$.

It follows that $p'(x) \leq 0$ and $p''(x) \geq 0$, which ensures that the second-order condition for a maximum is always satisfied because $x > o - \phi$:

$$p''(x)x + 2p'(x) - p''(x)(o - \phi) < 0$$

We derive $\underline{x}$ and k using the requirement that the taker prefers accept over reject-take for $x = \underline{x}$:

$$\underline{x} \leq \mathrm{E}(o|x = \underline{x}) + \phi \Leftrightarrow \underline{x} \leq \frac{\underline{x} - \phi}{2} + \phi$$

$$\underline{x} \leq \phi$$

Because the taker strictly prefers accept over reject-breach for any $x < \phi$ it follows that

$$\underline{x} = \phi$$

$$k = e^{\frac{1}{2}}$$

The equilibrium in case (II) is restricted by $t \leq 2\phi \ln \frac{2\phi}{\phi - \psi} + \phi - \psi$ because without the restriction, no $p(x)$ can induce all owners with $o < t + \psi - \phi$ to make the separating demand $x = o + \phi$ instead of demanding $x > t + \psi$, inducing the taker to respect the entitlement. Owners only make the separating demand $x = o + \phi$ if they prefer the taker's mixed response to inducing the taker to reject and respect the entitlement:

$$p(x)(x-o)+\big(1-p(x)\big)(-\phi)\geq -\psi$$

$$p(o+\phi)(\phi)+\big(1-p(v+\phi)\big)(-\phi)\geq -\psi$$

We consider the marginal owner with valuation $\bar{o}$ who is just indifferent (and still chooses to make an offer $x=o+\phi$):

$$p(\bar{o}+\phi)(\phi)+\big(1-p(\bar{o}+\phi)\big)(-\phi)=-\psi$$

Knowing that $k=e^{\frac{1}{2}}$, we use $p(x)=e^{\frac{\phi-x}{2\phi}}$ to get

$$e^{\frac{-\bar{o}}{2\phi}}=\frac{\phi-\psi}{2\phi}\Leftrightarrow \bar{o}=2\phi\ln\frac{2\phi}{\phi-\psi}$$

To induce separation of all owners with $o\leq t+\psi-\phi$ it must be that $\bar{o}+\phi\geq t+\psi$. This only holds if $t\leq 2\phi\ln\frac{2\phi}{\phi-\psi}+\phi-\psi$.

(3) Case (III), $\bar{\mathbf{o}}+\boldsymbol{\phi}-\boldsymbol{\psi}<\boldsymbol{t}\leq\frac{\boldsymbol{H}+\bar{\mathbf{o}}}{\mathbf{2}}+\boldsymbol{\phi}-\boldsymbol{\psi}$: Separating equilibrium with mimicking owners and takers respecting the entitlement in case of demands $\boldsymbol{x}>\boldsymbol{t}+\boldsymbol{\psi}$

From case (II), we retain $\bar{o}=2\phi\ln\frac{2\phi}{\phi-\psi}$, the highest valuation for which the taker's mixed strategy can elicit a fully revealing demand. For the taker's valuations above the upper limit of case (II), that is, $t>\bar{o}+\phi-\psi$, there exist owners with valuation o such that $\bar{o}<o<t+2\psi$.

We start by showing that the taker's strategy as stated in Proposition 5 (III) is in equilibrium, given that owners with $o\in]\bar{o},t+2\psi]$ demand $x>t+\psi$ just like owners with $o>t+2\psi$. Note that this implies no demands $x\in]\bar{o}+\phi,t+\psi]$ are made in equilibrium. This permits a taker strategy prescribing reject-take for such demands. (Whether such a strategy survives reasonable refinements for out-of-equilibrium beliefs will be discussed subsequently.) We refer to the proof of Proposition 5 (II) for showing that the taker's mixed strategy to demands $x\leq\bar{o}+\phi$ is in equilibrium. It remains to show that the taker still prefers respecting the entitlement to take if she receives a demand $x>t+\psi$:

$$-\psi \geq t - E(v|x > t + \psi) - \phi = t - \frac{\bar{o} + H}{2} - \phi = t - \frac{2\phi \ln \frac{2\phi}{\phi - \psi} + H}{2} - \phi$$

$$t \leq \frac{H}{2} + \phi \ln \frac{2\phi}{\phi - \psi} + \phi - \psi$$

This gives us the upper limit of case (III).

Turning to the owner, owners with valuations $o \in]\bar{o}, t + 2\psi]$ are better off demanding $x > t + \psi$ than by making a demand $x \in]\bar{o} + \phi, t + \psi]$, because the owner believes the taker would respond with reject-take. For other owner valuations, the proof of Proposition 5 (II) continues to apply.

So far, we have not constrained the owner's strategy for demands $x \in]\bar{o} + \phi, t + \psi]$ that do not occur in equilibrium. However, equilibrium responses to off-equilibrium moves should be based on reasonable beliefs. In what follows, we use the intuitive criterion of Cho and Kreps (1987) and the D1 criterion of Banks and Sobel (1987). Applied to our setting, both criteria constrain the taker's beliefs if faced with an out-of-equilibrium demand. This can lead one to dismiss the taker's reject-take response to a deviating demand and induce certain owners to make such a demand, eliminating the respective equilibrium.

Intuitive Criterion. The intuitive criterion of Cho and Kreps (1987) requires the taker to believe that only owner types make a deviating demand whose payoffs are not dominated by their payoffs from following their equilibrium strategy; i.e., their equilibrium payoff is less than the highest possible payoff they could obtain from the out-of-equilibrium demand. Since the taker never desits after rejecting a demand $x < t + \psi$, we can restrict attention to his acceptance probability $p(x)$. Formally, for a owner of type o, an out-of-equilibrium demand x, and an equilibrium strategy $x^*(o)$, the set of acceptance probabilities q for which the owner is better off deviating is the following:

$$D(o,x) \coloneqq \{q \in [0,1] \mid \Pi_O\big(o, x^*(v)\big) \leq q(x - o) + (1 - q)(-\phi)\}$$

After observing a demand x, the intuitive criterion requires the taker to rule out all owner types for which $D(o,x)$ is empty. Remember that out-of-equilibrium demands x are in the interval $]\bar{o}+\phi, t+\psi]$.

For owners with $o \leq \bar{o}$, the set $D(o,x)$ is non-empty:

$$\Pi_O(o, x^*(v)) \leq q(x-o) + (1-q)(-\phi)$$

This inequality is easiest satisfied by setting $q = 1$ because $x > \bar{o} + \phi$:

$$p(o+\phi)(\phi) + \big(1 - p(v+\phi)\big)(-\phi) \leq x - o$$

The latter inequality always holds for off-equilibrium demands $x > \bar{o} + \phi$ from owners with $o \leq \bar{o}$.

For owners with $o > \bar{o}$, the equilibrium payoff is $\Pi_O(o, x^* > t + \psi) = -\psi$. The payoff from a deviating demand $x \in]\bar{o}+\phi, t+\psi]$ dominates the equilibrium payoff for any q that satisfies the following inequality:

$$\Pi_O(o, x^*(o)) \leq q(x-o) + (1-q)(-\phi)$$

$$-\psi \leq q(x-o) + (1-q)(-\phi)$$

$$o \leq x + \phi - \frac{\phi - \psi}{q}$$

Again, this inequality is least restrictive for $q = 1$, so that it can be satisfied for all owners with

$$o \leq x + \psi$$

Given that for off-equilibrium demands $x > \bar{o} + \phi$, the latter condition for owners $o > \bar{o}$ implies the former one for owners $o \leq \bar{o}$. It follows that if the taker observes a deviating demand, she believes owners with $o \leq x + \psi$ to make such a demand. The intuitive criterion further requires the taker to attach the same probability of a deviating demand x to all remaining owner types with a non-empty $D(o,x)$. For our equilibrium to survive the intuitive criterion, the taker must weakly prefer rejecting all deviating demands, i.e., $p(x) = 0$ for all $x \in]\bar{o}+\phi, t+\psi]$, to accepting with any strictly positive probability, given the belief so defined. This implies

$$t - x < t - E(v|v \leq x + \psi) - \phi$$

$$x > E(v|v \leq x + \psi) + \phi$$

$$x > \frac{x + \psi}{2} + \phi$$

$$x > 2\phi + \psi$$

This inequality is hardest to satisfy for x at the lower end of the interval $]\bar{o} + \phi, t + \psi]$:

$$\bar{o} + \phi > 2\phi + \psi$$

$$2\phi \ln \frac{2\phi}{\phi - \psi} > \phi + \psi$$

As this expression is always true, the equilibrium is robust to the intuitive criterion.

D1 Criterion. The D1 criterion states that the taker believes a deviating move to come from an owner type who is "most likely" to make it; the "most likely" types are the ones who can benefit from the deviation for the largest set of taker responses, compared to their equilibrium strategy. In contrast to the intuitive criterion, the taker's belief does not include all owner types who can potentially improve their payoff.

We now denote as $D(o, x)$ the set of taker responses q for which an owner with valuation o is strictly better off making a deviating demand:

$$D(o, x) \coloneqq \{q \in [0,1] \mid \Pi_O\big(o, x^*(o)\big) < q(x - o) + (1 - q)(-\phi)\}$$

The corresponding set $D^0(o, x)$ for the owner being indifferent between the equilibrium and the out-of-equilibrium demand is:

$$D^0(o, x) \coloneqq \{q \in [0,1] \mid \Pi_O\big(o, x^*(o)\big) = q(x - o) + (1 - q)(-\phi)\}$$

The D1 criterion provides that if there exists a type of owner o' such that $D(o, x) \cup D^0(o, x) \subset D(v', x)$, then type o can "be pruned from the tree", that is, the taker, upon observing o, assigns zero probability to the deviating owner being of type o.

The taker ascribes positive probability only to the set of types of owners that cannot be eliminated in this way.

No owner can be better off making an off-equilibrium demand $x < o - \phi$. Since we know from the intuitive criterion that only owners with $o \leq x + \psi$ can be better off deviating, we can also rule out $x = o - \phi$. For the remaining case $x > o - \phi$, the deviation payoff $q(x - o) + (1 - q)(-\phi)$ is strictly increasing in q. Denote as $q^0(x, o)$ the taker's acceptance probability for an off-equilibrium demand x for which an owner of type o is indifferent between her equilibrium strategy $x^*(o)$ and the deviating demand x. $q^0(x, o)$ is the probability threshold above which the respective owner is strictly better off deviating.

$q^0(x, o)$ is defined by

$$\Pi_O\big(o, x^*(o)\big) = q^0(x, o)(x - o) + \big(1 - q^0(x, o)\big)(-\phi)$$

$$q^0(x, o) = \frac{\Pi_O\big(o, x^*(o)\big) + \phi}{x - o + \phi}$$

For $o \leq \bar{o}$, this becomes

$$q^0(x, o) = \frac{p(o + \phi)(\phi) + \big(1 - p(o + \phi)\big)(-\phi) + \phi}{x - o + \phi}$$

$$q^0(x, o) = \frac{p(o + \phi)2\phi}{x - o + \phi}$$

Differentiating for o gives us

$$\frac{\mathrm{d}\, q^0(x, o)}{\mathrm{d\, o}} = \frac{p'(o + \phi)2\phi}{x - o + \phi} + \frac{p(o + \phi)2\phi}{(x - o + \phi)^2}$$

Using $p(x)$ from above, we get

$$\frac{\mathrm{d}\, q^0(x, o)}{\mathrm{d\, o}} = \frac{-\frac{1}{2\phi}e^{-\frac{o}{2\phi}}2\phi}{x - o + \phi} + \frac{e^{-\frac{o}{2\phi}}2\phi}{(x - o + \phi)^2}$$

$$\frac{\mathrm{d}\, q^0(x, o)}{\mathrm{d\, o}} = e^{-\frac{o}{2\phi}}\frac{o + \phi - x}{(x - o + \phi)^2}$$

Because out-of-equilibrium demands satisfy $x \geq \bar{o} + \phi$ and we are considering owners $o \leq \bar{o}$, the derivative is negative: As o increases, the threshold acceptance probability $q^0(x, o)$ declines. Owners with higher valuation are more "likely" in the sense of the D1 criterion to deviate to any given off-equilibrium demand x. This implies that, among the owners with $o \leq \bar{o}$, we can confine attention to the single owner with $o = \bar{o}$.

For owners $o > \bar{o}$, the threshold probability is

$$q^0(x, o) = \frac{-\psi + \phi}{x - o + \phi}$$

The derivative is $\frac{\mathrm{d}\, q^0(x,o)}{\mathrm{d\ o}} = \frac{\phi - \psi}{(x - o + \phi)^2}$

which is always positive. Thus, we can restrict attention to an owner with $o = \lim_{\epsilon \to 0} \bar{o} + \epsilon$ with equilibrium payoff $\bar{o} - \psi$. By construction, this exactly equals the expected equilibrium payoff for owner type $o = \bar{o}$.

As a consequence, the taker's belief under the D1 criterion is that he faces either an owner with $o = \bar{o}$ or one with $o = \lim_{\epsilon \to 0} \bar{o} + \epsilon$. Sticking to her equilibrium strategy of reject-take therefore costs the taker $\bar{o} + \phi$ which is less than accepting an out-of-equilibrium demand $x \in]\bar{o} + \phi, t + \psi]$. The D1 criterion is hence satisfied.

(4) Case (IV), $\boldsymbol{\frac{H+\bar{o}}{2} + \phi - \psi < t \leq}$ H: Separating equilibrium with mimicking owners and taker playing mixed strategy for demands $\boldsymbol{x > t + \psi}$

The equilibrium of case (IV) differs from the one in case (III) in that the taker responds to a demand $x > t + \psi$ with a mixed strategy between respecting the entitlement and appropriating it. This lowers the owner's payoff from making such a high demand and thereby increases the highest owner type that makes a separating demand. Denote this higher threshold $\bar{\bar{o}}$. Owners with $o \leq \bar{\bar{o}}$ make a separating demand $x = o + \phi$. Owners with $o > \bar{\bar{o}}$ demand $x > t + \psi$. Demands $x \in]\bar{\bar{o}} + \phi, t + \psi]$ do not occur in equilibrium.

To determine $\bar{\bar{o}}$, note first that a strategy mix of accept-take with acceptance probability $p(x) = e^{\frac{-o}{2\phi}}$ places no limitation on $\bar{\bar{o}}$. In case (II) and (III), the relevant constraint came from the owner's alternative strategy to demand $x > t + \psi$, thereby forcing the taker to respect the entitlement. The equilibrium in case (IV) instead requires the taker to randomize between take and respecting if faced with a high demand $x > t + \psi$. To do this, she has to be indifferent.

$$t - E(o|x > t + \psi) - \phi = -\psi$$

Since $\bar{\bar{o}}$ is the relevant cutoff for owner types, this implies

$$\frac{H + \bar{\bar{o}}}{2} + \phi = t + \psi$$

$$\bar{\bar{o}} = 2t + 2\psi - 2\phi - H$$

Given her indifference, the taker can use an equilibrium probability π of respecting the entitlement in response to a high demand $x > t + \psi$; with probability $1 - \pi$, she takes. The owner's expected payoff from a high demand is $\Pi_O(x) = \pi(-\psi) + (1 - \pi)(-\phi)$. To determine π, we consider the marginal owner with valuation $\bar{\bar{o}}$. Because $\bar{\bar{o}}$ is the threshold, the marginal owner must be just indifferent between the separating demand $x = o + \phi$ and the high demand $x > t + \psi$. To the separating demand $x = o + \phi$, the taker responds as in case (II) and (III) with randomizing accept-take with acceptance probability $p(x)$. For the marginal owner to be indifferent, this gives us:

$$p(\bar{\bar{o}} + \phi)(\phi) + \left(1 - p(\bar{\bar{o}} + \phi)\right)(-\phi) = \pi(-\psi) + (1 - \pi)(-\phi)$$

$$\pi = \frac{2p(\bar{\bar{o}} + \phi)\phi}{\phi - \psi}$$

$$\pi = \frac{2\phi}{\phi - \psi} e^{-\frac{\bar{\bar{o}}}{2\phi}}$$

Plugging in $\bar{\bar{o}}$ yields

$$\pi = \frac{2\phi}{\phi - \psi} e^{-\frac{2t + 2\psi - 2\phi - H}{2\phi}}$$

For the taker to randomize, it must be that $\pi > 0$, which is always satisfied. More interestingly, $\pi < 1$ implies

$$t > \frac{H}{2} + \phi \, ln \frac{2\phi}{\phi - \psi} + \phi - \psi = \frac{H + \bar{o}}{2} + \phi - \psi$$

giving us the lower bound of case (IV).

In the course of determining $\bar{\bar{o}}$, we have already established that the taker's mixed strategy for demands $x > t + \psi$ is in equilibrium. We know her mixed-strategy response to separating demands $x \leq \bar{\bar{o}} + \phi$ to be in equilibrium from case (II) and case (III). In deriving π, we have implicitly shown that owners with $o \leq \bar{\bar{o}}$ will not make a high demand $x > t + \psi$, and owners with $o > \bar{\bar{o}}$ will not make a demand $x \leq \bar{\bar{o}} + \phi$. It remains to demonstrate that owners abstain from making off-equilibrium demands $x \in]\bar{\bar{o}} + \phi, t + \psi]$. Again, in a first step we merely prescribe the taker's equilibrium strategy reject-take in response to such demands. As a consequence, owners with $o \leq \bar{\bar{o}}$ are better off making a separating demand $x = o + \phi$ because $p(x)(x - o) + \big(1 - p(x)\big)(-\phi) > -\phi$. Owners with $o > \bar{\bar{o}}$ prefer a demand $x > t + \psi$ because it gives them $\pi(-\psi) + (1 - \pi)(-\phi) > -\phi$.

As in case (III), we also want to ensure that the equilibrium is robust to the intuitive criterion and the D1 criterion.

Intuititive criterion. For the definition of the intuitive criterion we refer to case (III). We start by finding owner types with a non-empty set of taker responses $D(o, x)$ to an out-of-equilibrium demand x under which the respective owner is better off than by playing his equilibrium strategy $x^*(o)$:

$$D(o, x) \coloneqq \{q \in [0, 1] \mid \; \Pi_O\big(o, x^*(o)\big) \leq q(x - o) + (1 - q)(-\phi)\}$$

Owners with $o \leq \bar{\bar{o}}$ are clearly better off with a taker response of always accepting, that is, $q = 1$:

$$p(o + \phi)(\phi) + \big(1 - p(o + \phi)\big)(-\phi) \leq x - o$$

This clearly holds for out-of-equilibrium demands $x \in]\bar{\bar{o}} + \phi, t + \psi]$. For all of these owners the set $D(o, x)$ is non-empty.

As to owners with $o > \bar{\bar{o}}$, the equilibrium payoff is $\Pi_O(o, x^* > t + \psi) = \pi(-\psi) + (1 - \pi)(-\phi)$. $D(o, x)$ is non-empty for these owners if

$$\pi(-\psi) + (1 - \pi)(-\phi) \leq q(x - o) + (1 - q)(-\phi)$$

$$\pi(\phi - \psi) \leq q(x - o + \phi) \tag{2}$$

This inequality is easiest to satisfy with $q = 1$, hence $D(o, x)$ is non-empty if

$$o \leq x + \phi - \pi(\phi - \psi)$$

Denote the corresponding cutoff $\hat{o} = x + \phi - \pi(\phi - \psi)$. $\hat{v}$ is the relevant cutoff if $\hat{o} > \bar{\bar{o}}$:

$$\bar{\bar{o}} < x + \phi - \pi(\phi - \psi)$$

This inequality is hardest to satisfy for the smallest off-equilibrium $x = \bar{\bar{o}} + \phi$, giving us

$$-2\phi < -\pi(\phi - \psi)$$

Inserting $\pi = \frac{2\phi}{\phi - \psi} e^{-\frac{\bar{\bar{o}}}{2\phi}}$ and simplifying yields

$$1 > e^{-\frac{\bar{\bar{o}}}{2\phi}}$$

$$1 > e^{-\frac{2t + 2\psi - 2\phi - H}{2\phi}}$$

$$0 < 2t + 2\psi - 2\phi - H$$

Plugging in the lower boundary of case (IV), $t = \frac{H}{2} + \phi\, ln \frac{2\phi}{\phi - \psi} + \phi - \psi$, we arrive at

$$0 < 2\,\phi\, ln \frac{2\phi}{\phi - \psi}$$

which clearly holds. Hence, $\hat{o}$ is the relevant cutoff. When the taker observes a deviating demand, she believes that it comes from an owner with $o \leq \hat{o}$. The equilibrium then survives the intuitive criterion if

$$t - x \leq t - E(o|\, o \leq \hat{o}) - \phi$$

$$x \geq E(o|\, o \leq \hat{o}) + \phi$$

$$x \geq \frac{x + \phi - \pi(\phi - \psi)}{2} + \phi$$

$$x \geq 3\phi - \pi(\phi - \psi)$$

This inequality is hardest to satisfy with the lowest off-equilibrium demand $x = \bar{\bar{o}} + \phi$:

$$2t + 2\psi - 2\phi - H \geq 2\phi - \pi(\phi - \psi)$$

$$2t + 2\psi \geq 4\phi + H - \pi(\phi - \psi)$$

Using $\pi = \frac{2\phi}{\phi-\psi} e^{-\frac{2t+2\psi-2\phi-H}{2\phi}}$ gives us

$$t + \psi \geq 2\phi + \frac{H}{2} - \phi e^{-\frac{2t+2\psi-2\phi-H}{2\phi}}$$

$$t + \phi e^{-\frac{2t+2\psi-2\phi-H}{2\phi}} \geq 2\phi + \frac{H}{2} - \psi \tag{3}$$

To establish that inequality (3) always holds, we differentiate the left hand side for t and show that the derivative is positive:

$$1 + \phi\left(-\frac{1}{\phi}\right) e^{-\frac{2t+2\psi-2\phi-H}{2\phi}} > 0$$

$$e^{\frac{2t+2\psi-2\phi-H}{2\phi}} > 1$$

$$2t + 2\psi - 2\phi - H > 0$$

Inserting the smallest t in the range of case (IV), $t = \frac{H}{2} + \phi\, ln \frac{2\phi}{\phi-\psi} + \phi - \psi$:

$$2\phi \ln \frac{2\phi}{\phi - \psi} > 0$$

which is true.

Given that the derivative of the left hand side of inequality (3) is positive, we insert the minimum $t = \frac{H}{2} + \phi\, ln \frac{2\phi}{\phi - \psi} + \phi - \psi$ in inequality (3):

$$\frac{H}{2} + \phi \ln \frac{2\phi}{\phi - \psi} + \phi - \psi + \frac{\phi - \psi}{2} \geq 2\phi + \frac{H}{2} - \psi$$

$$2 \ln \frac{2\phi}{\phi - \psi} \geq 1 + \frac{\psi}{\phi}$$

$$2\phi \ln \frac{2\phi}{\phi - \psi} \geq \phi + \psi$$

Differentiating the left hand side for ψ yields $\frac{2\phi}{\phi - \psi}$, which exceeds the corresponding derivative of the right hand side. It follows that, since the inequality holds for $\psi = 0$, it holds throughout. Hence, the equilibrium satisfies the intuitive criterion.

D1 criterion. We know from the intuitive criterion that the most likely deviator must be an owner with either $o \leq \bar{\bar{o}}$ or $o \in]\bar{\bar{o}}, \hat{o}]$. As to the former group, the same reasoning as for case (III) leads us to focus on the marginal owner with $o = \bar{\bar{o}}$. If this were indeed the most likely deviating owner, the taker would still stick to her equilibrium strategy of $q = 0$ if

$$t - \bar{\bar{o}} - \phi > q(t - x) + (1 - q)(t - \bar{\bar{o}} - \phi)$$

$$\bar{\bar{o}} + \phi < qx + (1 - q)(\bar{\bar{o}} + \phi) \tag{4}$$

$$0 < q(x - \bar{\bar{o}} - \phi)$$

$$\bar{\bar{o}} + \phi < x$$

which is always true because off-equilibrium demands exceed $\bar{\bar{o}} + \phi$.

As to the latter group of owners with $o \in]\bar{\bar{o}}, \hat{o}]$, they benefit from an off-equilibrium demand $x \in]\bar{\bar{o}} + \phi, t + \psi]$ if inequality (2) is satisfied. Rearranging inequality (2) gives us

$$o \leq x + \phi - \frac{\pi(\phi - \psi)}{q}$$

This inequality holds for a greater range of q if o is at the lower bound of the interval $]\bar{\bar{o}}, \hat{o}]$. Therefore, the most likely owner to deviate from this interval has a valuation $o = \lim_{\epsilon \to 0} \bar{\bar{o}} + \epsilon$. If this owner turned out to be the most likely deviator in the sense of the D1 criterion, the condition for the taker to play her equilibrium strategy remains the one in inequality (4), which is always satisfied. Without determining whether the owner with $o = \bar{\bar{o}}$ or with $o = \lim_{\epsilon \to 0} \bar{\bar{o}} + \epsilon$ most likely makes an out-of-equilibrium demand, the taker's equilibrium response is robust.

c) Proposition 6. Welfare comparison of expectation damages and specific performance

In the signaling model the property rule is superior. It always leads to efficient agreements whereas with a liability rule, the parties' abilities to conclude efficient agreements depends on the taker's valuation.

The owner's expected payoff under the property rule is

$$\mathrm{E}_o(\Pi_{O\,PR}) = \min\left(\frac{t+2\psi}{H}, 1\right)(t+\psi) + \left(1 - \min\left(\frac{t+2\psi}{H}, 1\right)\right)\left(\frac{t+2\psi+H}{2} - \psi\right)$$

Thus, if $t \leq H - 2\psi$ the owner's payoff is

$$\mathrm{E}_o(\Pi_{O\,PR}) = \frac{H}{2} + \frac{t^2 + 4\psi t + 4\psi^2 - 2H\psi}{2H}$$

In case $t > H - 2\psi$ the owner's payoff is $\mathrm{E}_o(\Pi_{O\,PR}) = t + \psi$.

The taker's expected payoff is

$$\Pi_{T\,SP} = -\psi$$

The total welfare, for the case $t \leq H - 2\psi$, is

$$\Pi_{PR} = \frac{H}{2} + \frac{t^2 + 4\psi t + 4\psi^2 - 2H\psi}{2H} - \psi$$

$$\Pi_{PR} = \frac{H}{2} + \frac{(t+2\psi)^2 - 4H\psi}{2H} \tag{5}$$

If $t > H - 2\psi$ the total welfare is $\Pi_{PR} = t$.

Insofar as equilibrium strategies under the liability rule equal those under the property rule (Proposition 5 (I) with taker's valuation of $t \leq 2\phi$), surplus under the two remedies is the same.

All other cases require closer inspection.

(1) Welfare comparison for Case (II)

For Proposition 5 (II), that is, $2\phi < t \leq 2\phi \ln \frac{2\phi}{\phi-\psi} + \phi - \psi$, the owner's expected payoff under the liability rule is

$$\mathrm{E}_o(\Pi_{O\,LRII}) = \frac{t+\psi-\phi}{H}\left(\frac{t+2\psi-\phi}{2} + E(2p(o+\phi)\phi - \phi | o \leq t+\psi-\phi)\right) + \left(1 - \frac{t+\psi-\phi}{H}\right)\left(\frac{t+2\psi-\phi+H}{2} - \psi\right)$$

$$\mathrm{E}_o(\Pi_{O\,LRII}) = \frac{t+\psi-\phi}{H}\left(\frac{t+2\psi-\phi}{2} - \phi + 2\phi\frac{\int_0^{t+\psi-\phi} e^{-\frac{o}{2\phi}}\,do}{t+\psi-\phi}\right) + \left(1 - \frac{t+\psi-\phi}{H}\right)\left(\frac{t+2\psi-\phi+H}{2} - \psi\right)$$

$$\mathrm{E}_o(\Pi_{O\,LRII}) = \frac{t+\psi-\phi}{H}\left(\frac{t+2\psi-\phi}{2} - \phi + 4\phi^2\frac{1-e^{-\frac{t+\psi-\phi}{2\phi}}}{t+\psi-\phi}\right) + \left(1 - \frac{t+\psi-\phi}{H}\right)\left(\frac{t+2\psi-\phi+H}{2} - \psi\right)$$

$$\mathrm{E}_o(\Pi_{O\,LRII}) = \frac{H}{2} + \frac{(5-4e^{-\frac{t-\phi+\psi}{2\phi}})\phi^2 - 2\phi\psi + \psi(-H+\psi) + t(-\phi+\psi)}{H}$$

The taker's payoff is

$$\mathrm{E}_o(\Pi_{T\,LRII}) = \left(\frac{t+\psi-\phi}{H}\right)\left(t - \frac{t+\psi-\phi}{2} - \phi\right) + \left(1 - \frac{t+\psi-\phi}{H}\right)(-\psi)$$

$$\mathrm{E}_o(\Pi_{T\,LRII}) = \frac{t^2 - 2t\phi + \phi^2 - 2H\psi + 2t\psi - 2\phi\psi + \psi^2}{2H}$$

$$\mathrm{E}_o(\Pi_{T\,LRII}) = \frac{(t-\phi+\psi)^2 - 2H\psi}{2H}$$

Thus, the total welfare is

$$\Pi_{LRII} = \frac{H}{2} + \frac{(5-4e^{-\frac{t-\phi+\psi}{2\phi}})\phi^2 - 2\phi\psi + \psi(-H+\psi) + t(-\phi+\psi)}{H} + \frac{(t-\phi+\psi)^2 - 2H\psi}{2H}$$

$$\Pi_{LRII} = \frac{H}{2} + \frac{2(5-4e^{-\frac{t-\phi+\psi}{2\phi}})\phi^2 - 4\phi\psi + 2\psi(-H+\psi) + 2t(-\phi+\psi)}{2H} + \frac{(t-\phi+\psi)^2 - 2H\psi}{2H}$$

$$\Pi_{LRII} = \frac{H}{2} + \frac{2\left(5-4e^{-\frac{t-\phi+\psi}{2\phi}}\right)\phi^2 - 4\phi\psi + 2\psi^2 + 2t(-\phi+\psi) + (t-\phi+\psi)^2 - 4H\psi}{2H}$$

The property rule is more efficient if

$$\Pi_{PR} > \Pi_{LRII}$$

$$\frac{H}{2} + \frac{(t+2\psi)^2 - 4H\psi}{2H} > \frac{H}{2} + \frac{2\left(5-4e^{-\frac{t-\phi+\psi}{2\phi}}\right)\phi^2 - 4\phi\psi + 2\psi^2 + 2t(-\phi+\psi) + (t-\phi+\psi)^2 - 4H\psi}{2H}$$

Subtracting $\frac{H}{2}$ and multiplying by $2H$ and expanding both sides yields

$$t^2 + 4\psi t + 4\psi^2$$
$$> t^2 - 2t\phi + \phi^2 + 2t\psi - 2\phi\psi + \psi^2 + 2\left(5 - 4e^{-\frac{t-\phi+\psi}{2\phi}}\right)\phi^2$$
$$- 4\phi\psi + 2\psi^2 + 2t(-\phi + \psi)$$

Simplifying, we arrive at

$$\psi^2 > -4t\phi + \phi^2\left(11 - 8e^{-\frac{t-\phi+\psi}{2\phi}}\right) - 6\phi\psi$$

Because $t > 2\phi$, the second term on the right hand side is at most $-4(2\phi)\phi = -8\phi^2$. The above inequality thus is satisfied if the following, more restrictive inequality holds

$$\psi^2 > \phi^2\left(3 - 8e^{-\frac{t-\phi+\psi}{2\phi}}\right) - 6\phi\psi$$

Note that $8e^{-\frac{t-\phi+\psi}{2\phi}}$ is decreasing in t. Given that $t \leq 2\phi \ln\frac{2\phi}{\phi-\psi} + \phi - \psi$, the minimum is at $8\frac{\phi-\psi}{2\phi}$. Plugging this into the latter inequality gives us

$$\psi^2 > \phi^2\left(3 - 8\frac{\phi - \psi}{2\phi}\right) - 6\phi\psi$$

$$\psi^2 > 3\phi^2 - 4\phi^2 - 2\phi\psi$$

which is true. Hence, the property rule is more efficient than the liability rule.

(2) Welfare comparison for Case (III)

Case (III) obtains for $2\phi \ln\frac{2\phi}{\phi-\psi} + \phi - \psi < t \leq \frac{H}{2} + \phi\, ln\frac{2\phi}{\phi-\psi} + \phi - \psi$. The owner's expected payoff is

$$\mathrm{E}_o(\Pi_{O\ LRIII}) = \frac{\bar{o}}{H}(\frac{\bar{o}}{2} + E(2p(o+\phi)\phi - \phi | v \leq \bar{o})) + \left(1 - \frac{\bar{o}}{H}\right)\left(\frac{\bar{o} + H}{2} - \psi\right)$$

$$\mathrm{E}_o(\Pi_{O\ LRIII}) = \frac{\bar{o}}{H}\left(\frac{\bar{o}}{2} - \phi + 2\phi\frac{\int_0^{\bar{o}} e^{-\frac{o}{2\phi}}\, do}{\bar{o}}\right) + \left(1 - \frac{\bar{o}}{H}\right)\left(\frac{\bar{o} + H}{2} - \psi\right)$$

$$\mathrm{E}_o(\Pi_{O\,LRIII}) = \frac{\bar{o}}{H}\left(\frac{\bar{o}}{2} - \phi + 4\phi^2 \frac{1 - e^{-\frac{\bar{o}}{2\phi}}}{\bar{o}}\right) + \left(1 - \frac{\bar{o}}{H}\right)\left(\frac{\bar{o} + H}{2} - \psi\right)$$

$$\mathrm{E}_o(\Pi_{O\,LRIII}) = \frac{H}{2} - \psi - \frac{\bar{o}}{H}(\phi - \psi) + 4\phi^2 \frac{1 - e^{-\frac{\bar{o}}{2\phi}}}{H}$$

Using $\bar{o} = 2\phi \ln \frac{2\phi}{\phi - \psi}$

$$\mathrm{E}_o(\Pi_{O\,LRIII}) = \frac{H}{2} - \psi - \frac{\bar{o}}{H}(\phi - \psi) + \frac{4\phi^2(1 - \frac{\phi - \psi}{2\phi})}{H}$$

$$\mathrm{E}_o(\Pi_{O\,LRIII}) = \frac{H}{2} - \psi - \frac{\bar{o}}{H}(\phi - \psi) + \frac{4\phi^2(1 - \frac{\phi - \psi}{2\phi})}{H}$$

$$\mathrm{E}_o(\Pi_{O\,LRIII}) = \frac{H}{2} - \psi - \frac{\bar{o}}{H}(\phi - \psi) + \frac{2(\phi^2 + \phi\psi)}{H}$$

The taker's expected payoff is

$$\mathrm{E}_o(\Pi_{T\,LRIII}) = -\psi - \frac{\bar{o}^2}{2H} + \frac{\bar{o}}{H}(t - \phi + \psi)$$

It follows for the total welfare

$$\Pi_{LRIII} = \frac{H}{2} - \psi - \frac{\bar{o}}{H}(\phi - \psi) + \frac{2(\phi^2 + \phi\psi)}{H} - \psi - \frac{\bar{o}^2}{2H} + \frac{\bar{o}}{H}(t - \phi + \psi)$$

$$\Pi_{LRIII} = \frac{H}{2} - \psi - \frac{\bar{o}}{H}(\phi - \psi) + \frac{2(\phi^2 + \phi\psi)}{H} - \psi - \frac{\bar{o}^2}{2H} + \frac{\bar{o}}{H}(t - \phi + \psi)$$

$$\Pi_{LRIII} = \frac{H}{2} + \frac{-\bar{o}^2 + 2\bar{o}(t - 2\phi + 2\psi) + 4(\phi^2 - H\psi + \phi\psi)}{2H}$$

The property rule is more efficient because

$$\Pi_{PR} > \Pi_{LRIII}$$

$$\frac{H}{2}+\frac{(t+2\psi)^2-4H\psi}{2H}>\frac{H}{2}+\frac{-\bar{o}^2+2\bar{o}(t-2\phi+2\psi)+4(\phi^2-H\psi+\phi\psi)}{2H}$$

$$t^2-4H\psi+4t\psi+4\psi^2>-\bar{o}^2+2\bar{o}t-4\bar{o}\phi+4\phi^2-4H\psi+4\bar{o}\psi+4\phi\psi$$

$$t^2+4t\psi+4\psi^2>-\bar{o}^2+2\bar{o}t-4\bar{o}\phi+4\phi^2+4\bar{o}\psi+4\phi\psi$$

$$t^2+4t\psi+4\psi^2+\bar{o}^2-2\bar{o}t+4\bar{o}\phi-4\phi^2-4\bar{o}\psi-4\phi\psi>0$$

$$(t-\bar{o})^2+4\phi(\bar{o}-\phi)+4\psi(t-\bar{o}-\phi+\psi)>0$$

The first term is positive. Because $\bar{o}=2\phi\ln\frac{2\phi}{\phi-\psi}$, the second term ist strictly positive. Because $t>2\phi\ln\frac{2\phi}{\phi-\psi}+\phi-\psi$ in case (III), the inequality holds if the third term is positive for $t=2\phi\ln\frac{2\phi}{\phi-\psi}+\phi-\psi$, which it is.

(3) Welfare comparison for Case (IV)

For case (IV) with $\frac{H}{2}+\phi\, ln\frac{2\phi}{\phi-\psi}+\phi-\psi<t\leq H$, owner's expected payoff under the liability rule is

$$\mathrm{E}_o(\Pi_{O\ LRIV})=\frac{\bar{\bar{o}}}{H}\left(\frac{\bar{\bar{o}}}{2}-\phi+4\phi^2\frac{1-e^{-\frac{\bar{\bar{o}}}{2\phi}}}{\bar{\bar{o}}}\right)$$
$$+\left(1-\frac{\bar{\bar{o}}}{H}\right)\left(\frac{\bar{\bar{o}}+H}{2}+\pi(-\psi)+(1-\pi)(-\phi)\right)$$

$$\mathrm{E}_o(\Pi_{O\ LRIV})=\frac{\bar{\bar{o}}}{H}\left(\frac{\bar{\bar{o}}}{2}-\phi+4\phi^2\frac{1-e^{-\frac{\bar{\bar{o}}}{2\phi}}}{\bar{\bar{o}}}\right)+\left(1-\frac{\bar{\bar{o}}}{H}\right)\left(\frac{\bar{\bar{o}}+H}{2}-\phi+\pi(\phi-\psi)\right)$$

$$\mathrm{E}_o(\Pi_{O\ LRIV})=\frac{H}{2}-\phi\frac{\bar{\bar{o}}}{H}+4\phi^2\frac{1-e^{-\frac{\bar{\bar{o}}}{2\phi}}}{H}+\left(1-\frac{\bar{\bar{o}}}{H}\right)\big(-\phi+\pi(\phi-\psi)\big)$$

$$\mathrm{E}_o(\Pi_{O\ LRIV}) = \frac{H}{2} + 4\phi^2 \frac{1-e^{-\frac{\bar{\bar{o}}}{2\phi}}}{H} - \phi + \left(1-\frac{\bar{\bar{o}}}{H}\right)\left(\pi(\phi-\psi)\right)$$

Inserting $\pi = \frac{2\phi}{\phi-\psi} e^{-\frac{\bar{\bar{o}}}{2\phi}}$ gives us

$$\mathrm{E}_o(\Pi_{O\ LRIV}) = \frac{H}{2} + 4\phi^2 \frac{1-e^{-\frac{\bar{\bar{o}}}{2\phi}}}{H} - \phi + \left(1-\frac{\bar{\bar{o}}}{H}\right)\left(2\phi e^{-\frac{\bar{\bar{o}}}{2\phi}}\right)$$

The taker's expected payoff is

$$\mathrm{E}_o(\Pi_{T\ LRIV}) = \frac{\bar{\bar{o}}}{H}\left(t-\frac{\bar{\bar{o}}}{2}-\phi\right) + \left(1-\frac{\bar{\bar{o}}}{H}\right)\left(t-\frac{\bar{\bar{o}}+H}{2}-\phi-\pi\left(t+\psi-\phi-\frac{\bar{\bar{o}}+H}{2}\right)\right)$$

Plugging in $\bar{\bar{o}} = 2t+2\psi-2\phi-H$ provides

$$\mathrm{E}_o(\Pi_{T\ LRIV}) = \frac{\bar{\bar{o}}}{H}\left(\frac{H}{2}-\psi\right) + \left(1-\frac{\bar{\bar{o}}}{H}\right)(-\psi)$$

$$\mathrm{E}_o(\Pi_{T\ LRIV}) = \frac{\bar{\bar{o}}}{2} - \psi = t - \frac{H}{2} - \phi$$

The total welfare then is:

$$\Pi_{LRIV} = \frac{H}{2} + 4\phi^2 \frac{1-e^{-\frac{\bar{\bar{o}}}{2\phi}}}{H} - \phi + \left(1-\frac{\bar{\bar{o}}}{H}\right)\left(2\phi e^{-\frac{\bar{\bar{o}}}{2\phi}}\right) + \frac{\bar{\bar{o}}}{2} - \psi$$

$$\Pi_{LRIV} = \frac{H}{2} + \frac{8\phi^2 - 4e^{-\frac{\bar{\bar{o}}}{2\phi}}\phi(-H+\bar{\bar{o}}+2\phi) + H(\bar{\bar{o}}-2(\phi+\psi))}{2H}$$

$$\Pi_{LRIV} = \frac{H}{2} + \frac{8\phi^2 - 8\phi e^{-\frac{2t+2\psi-2\phi-H}{2\phi}}(-H+t+\psi) + H(2t-4\phi-H)}{2H}$$

The property rule is more efficient if

$$\Pi_{PR} > \Pi_{LRIV}$$

This holds for $t \leq H - 2\psi$:

$$\frac{H}{2} + \frac{(t+2\psi)^2 - 4H\psi}{2H} > \frac{H}{2} + \frac{8\phi^2 - 4e^{-\frac{\bar{\bar{o}}}{2\phi}}\phi(-H + \bar{\bar{o}} + 2\phi) + H(\bar{\bar{o}} - 2(\phi+\psi))}{2H}$$

Subtracting $\frac{H}{2}$ and multiplying by $2H$:

$$(t+2\psi)^2 - 4H\psi > 8\phi^2 - 4e^{-\frac{\bar{\bar{o}}}{2\phi}}\phi(-H + \bar{\bar{o}} + 2\phi) + H\big(\bar{\bar{o}} - 2(\phi+\psi)\big)$$

$$t^2 - 4H\psi + 4t\psi + 4\psi^2 > 8\phi^2 - 4e^{-\frac{\bar{\bar{o}}}{2\phi}}\phi(-H + \bar{\bar{o}} + 2\phi) + H\big(\bar{\bar{o}} - 2(\phi+\psi)\big)$$

$$t^2 + 4t\psi + 4\psi^2 > 8\phi^2 - 4e^{-\frac{\bar{\bar{o}}}{2\phi}}\phi(-H + \bar{\bar{o}} + 2\phi) + H\bar{\bar{o}} - 2H(\phi - \psi)$$

Inserting $\bar{\bar{o}} = 2t + 2\psi - 2\phi - H$

$$\begin{aligned} &t^2 + 4t\psi + 4\psi^2 \\ &\quad > 8\phi^2 - 4e^{-\frac{2t+2\psi-2\phi-H}{2\phi}}\phi(-H + 2t + 2\psi - 2\phi - H + 2\phi) + H(2t \\ &\quad + 2\psi - 2\phi - H) - 2H(\phi - \psi) \end{aligned}$$

$$\begin{aligned} &t^2 + 4t\psi + 4\psi^2 \\ &\quad > 8\phi^2 - 8e^{-\frac{2t+2\psi-2\phi-H}{2\phi}}\phi(-H + t + \psi) + H(2t + 2\psi - 2\phi - H) \\ &\quad - 2H(\phi - \psi) \end{aligned}$$

$$\begin{aligned} &t^2 - 2Ht + H^2 + 4t\psi + 4\psi^2 \\ &\quad > 8\phi^2 - 8e^{-\frac{2t+2\psi-2\phi-H}{2\phi}}\phi(-H + t + \psi) - 4H(\phi - \psi) \end{aligned}$$

$$(\mathrm{t} - H)^2 > (8\phi^2 - 8e^{-\frac{2t+2\psi-2\phi-H}{2\phi}}\phi(-H + t + \psi) - 4t\psi - 4\psi^2) - 4H(\phi - \psi)$$

The inequality holds because the right hand side will always be negative:

$$8\phi^2 - 8\phi e^{-\frac{2t+2\psi-2\phi-H}{2\phi}}(-H + t + \psi) - 4t\psi - 4\psi^2 - 4H(\phi - \psi) < 0$$

$$2\phi^2 - 2\phi e^{-\frac{2t+2\psi-2\phi-H}{2\phi}}(-H + t + \psi) - t\psi - \psi^2 - H(\phi - \psi) < 0$$

This equation has no interior minimum or maximum. For the first derivative in respect to t, we get:

$$-2e^{\frac{H-2(x-\phi+\psi)}{2\phi}}(H - t + \phi - \psi) - \psi$$

The derivative cannot be zero but will always be negative.

Therefore, the equation has its maximum at the lower bound of the interval for t, i.e. $t = \frac{H}{2} + \phi\, ln \frac{2\phi}{\phi-\psi} + \phi - \psi$.

$$2\phi^2 - 2\phi e^{-\frac{2\left(\frac{H}{2}+\phi\, ln\frac{2\phi}{\phi-\psi}+\phi-\psi\right)+2\psi-2\phi-H}{2\phi}}\left(-H + \left(\frac{H}{2} + \phi\, ln\frac{2\phi}{\phi-\psi} + \phi - \psi\right) + \psi\right) - \left(\frac{H}{2} + \phi\, ln\frac{2\phi}{\phi-\psi} + \phi - \psi\right)\psi - \psi^2 - H(\phi - \psi) < 0$$

$$2\phi^2 - 2\phi e^{-ln\frac{2\phi}{\phi-\psi}}\left(-\frac{H}{2} + \phi\, ln\frac{2\phi}{\phi-\psi} + \phi\right) - \left(\frac{H}{2} + \phi\, ln\frac{2\phi}{\phi-\psi} + \phi\right)\psi - H(\phi - \psi) < 0$$

$$2\phi^2 - 2\phi e^{-ln\frac{2\phi}{\phi-\psi}}\left(-\frac{H}{2} + \phi\, ln\frac{2\phi}{\phi-\psi} + \phi\right) - \left(\frac{H}{2} + \phi\, ln\frac{2\phi}{\phi-\psi} + \phi\right)\psi - H(\phi - \psi) < 0$$

$$2\phi^2 - (\phi - \psi)\left(-\frac{H}{2} + \phi\, ln\frac{2\phi}{\phi-\psi} + \phi\right) - \left(\frac{H}{2} + \phi\, ln\frac{2\phi}{\phi-\psi} + \phi\right)\psi - H(\phi - \psi) < 0$$

$$2\phi^2 - (\phi - \psi)\left(\phi\, ln\frac{2\phi}{\phi-\psi} + \phi\right) - \left(\frac{H}{2} + \phi\, ln\frac{2\phi}{\phi-\psi} + \phi\right)\psi - \frac{H}{2}(\phi - \psi) < 0$$

$$\phi^2 - \phi\left(\phi\, ln\frac{2\phi}{\phi-\psi}\right) - \left(\frac{H}{2}\right)\psi - \frac{H}{2}(\phi - \psi) < 0$$

$$\phi^2 - \phi\left(\phi\, ln\frac{2\phi}{\phi-\psi}\right) - \phi\frac{H}{2} < 0$$

$$\phi - \left(\phi \, ln \frac{2\phi}{\phi - \psi}\right) - \frac{H}{2} < 0$$

The inequality is hardest to satisfy for $\psi = 0$.

$$\phi - \phi(ln \, 2) < \frac{H}{2}$$

It follows that the equation always holds for $\phi \leq H$ which is true by assumption.

The property rule is also more efficient in case $t > H - 2\psi$.

Because we have shown that the payoff function of the property rule for lower values provides a higher total payoff for the whole range of values for t than the liability rule we only need to show that the total payoff under the property rule is higher than if we applied the payoff function of the property rule for lower values:

$$t > \frac{H}{2} + \frac{(t + 2\psi)^2 - 4H\psi}{2H}$$

$$2Ht - H^2 > (t + 2\psi)^2 - 4H\psi$$

$$t > H - 2\psi$$

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